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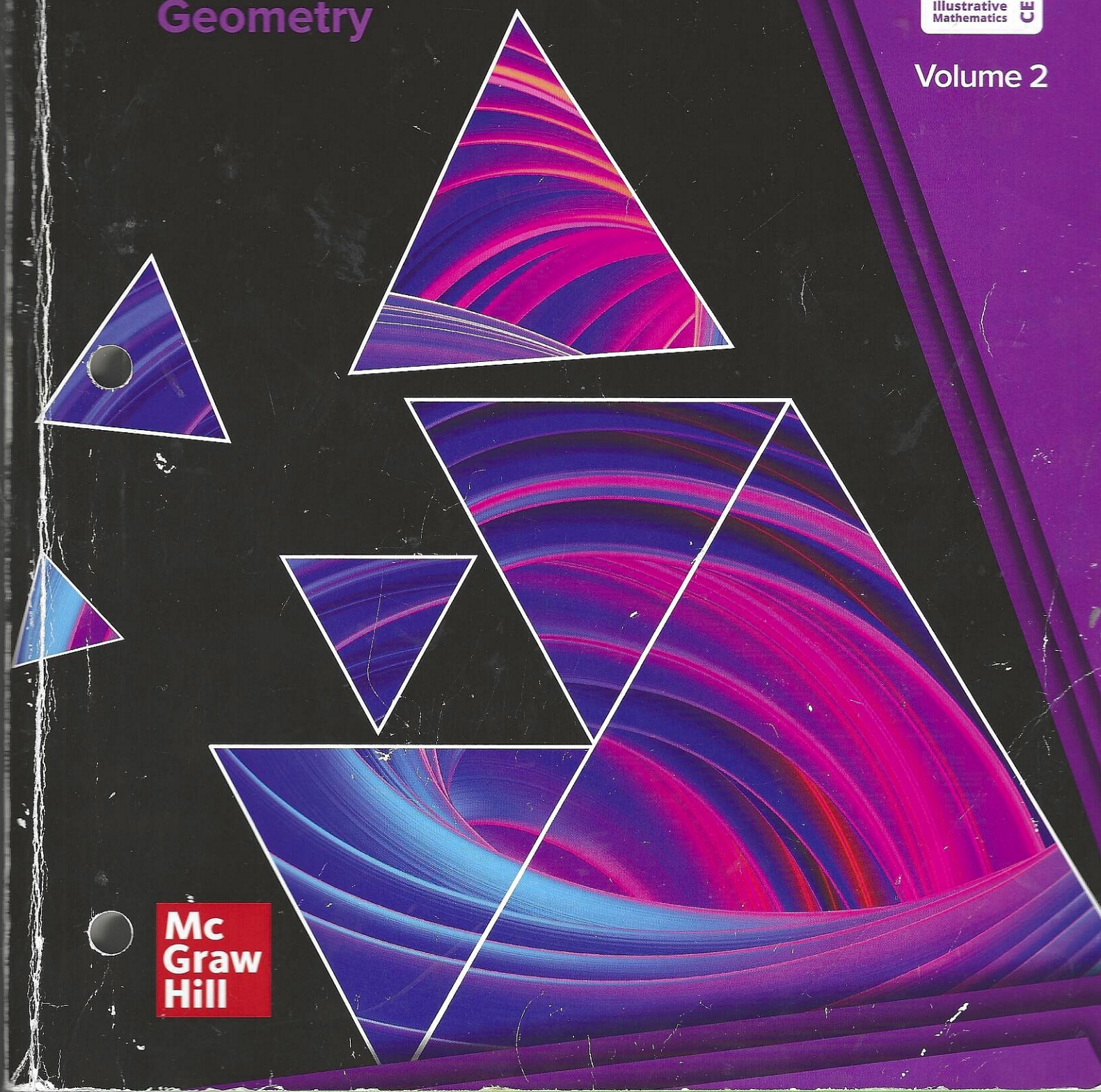
Volume 2



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Part 2 of 4–Unit 6

Volume 2

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Illustrative Mathematics®

Geometry

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Illustrative Mathematics is designed to provide opportunities for you to learn math by doing math. By engaging in problem-solving activities, you will make and test your own conclusions and try out different strategies as you learn math.

Let's get started!

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Unit 6

Coordinate Geometry

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A raindrop can produce ripples in a pond that illustrate dilations, a type of transformation. You will learn more about coordinate geometry in this unit.

Topics

- Transformations in the Plane
- Distances, Circles, and Parabolas
- Proving Geometric Theorems Algebraically
- Putting It All Together

Unit 6

Coordinate Geometry

Transformations in the Plane

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Lesson 6-1

Rigid Transformations in the Plane

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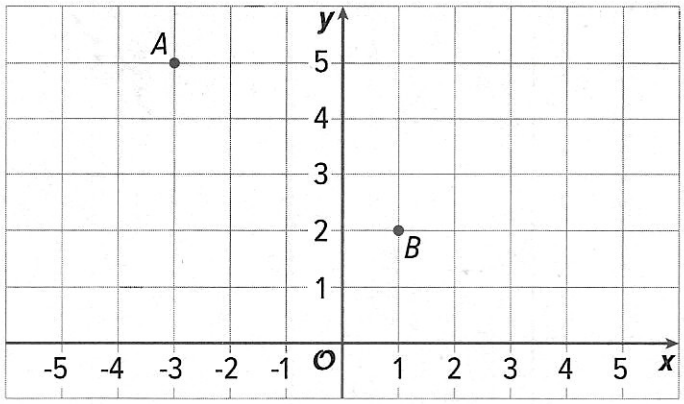
Learning Goal Let's try transformations with coordinates.

Warm Up

1.1 Traversing the Plane



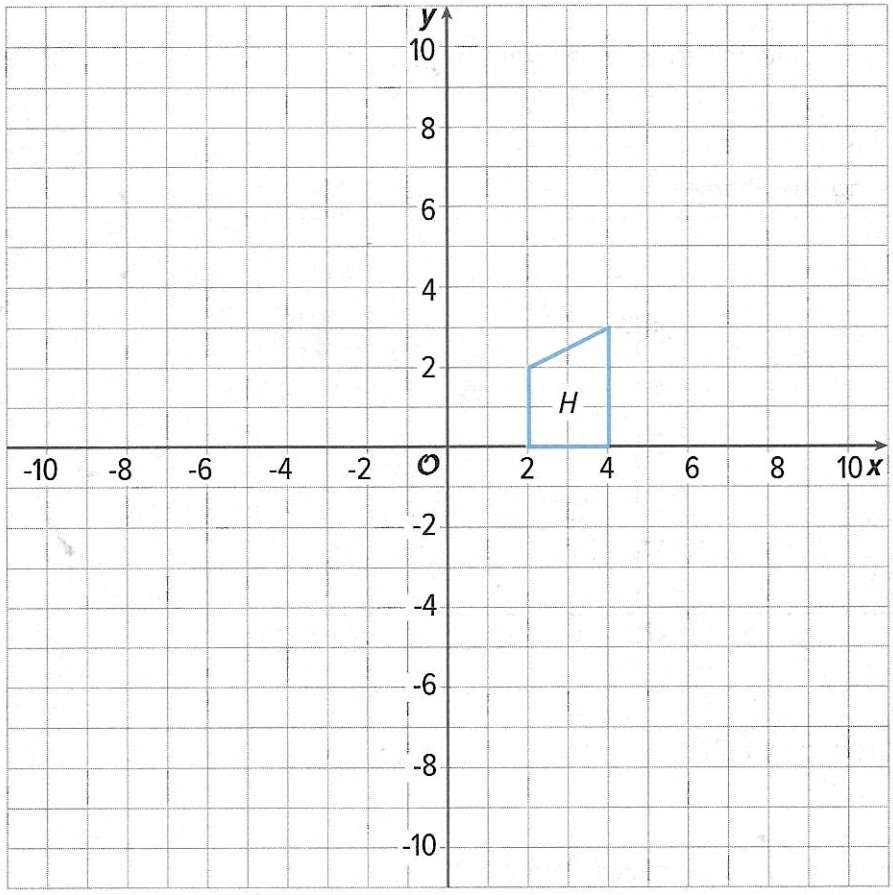
1. How far is point A from point B?

2. What transformations will take point A to point B?

Activity

1.2 Transforming with Coordinates

First, predict where each transformation will land. Next, carry out the transformation.



1. Rotate Figure *H* clockwise using center (2, 0) by 90 degrees. Translate the image by the directed line segment from (2,0) to (3, -4). Label the result *R*.
2. Reflect Figure *H* across the *y*-axis. Rotate the image counterclockwise using center (0, 0) by 90 degrees. Label the result *L*.

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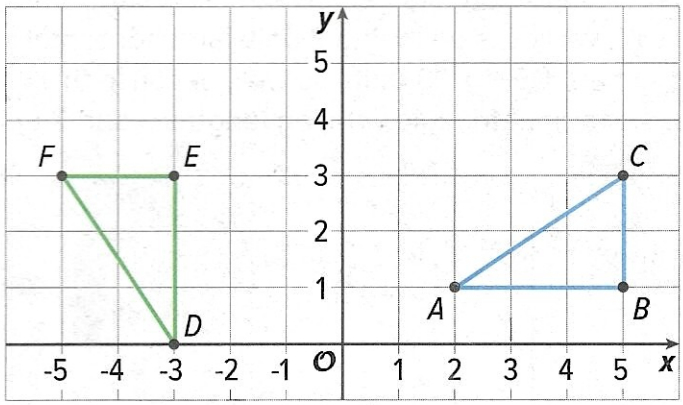
Activity

1.3 Congruent by Coordinates



1. Calculate the length of each side in triangles *ABC* and *DEF*.

2. Calculate the measure of each angle in triangles *ABC* and *DEF*.

3. The triangles are congruent. How do you know this is true?

4. Because the triangles are congruent, there must be a sequence of rigid motions that takes one to the other. Find a sequence of rigid motions that takes triangle *ABC* to triangle *DEF*.

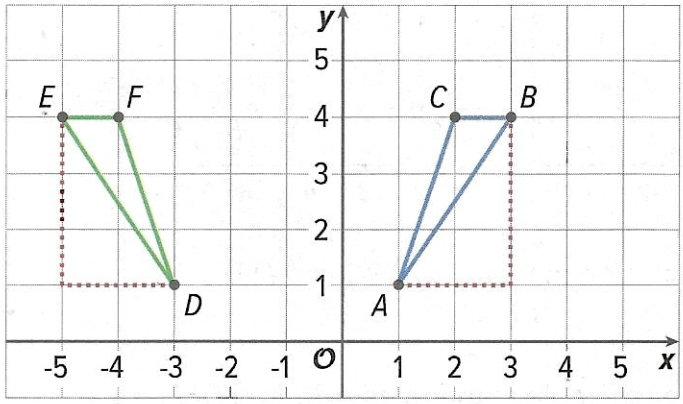
Are you ready for more?

What single transformation would take triangle *ABC* to triangle *DEF*?

Summary

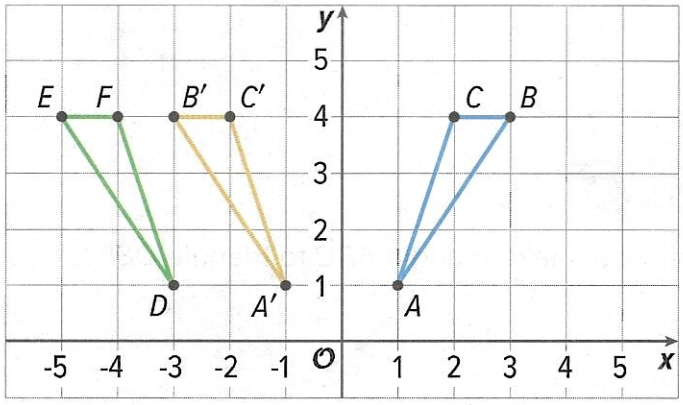
Rigid Transformations in the Plane

The triangles shown here look like they might be congruent. Since we know the coordinates of all the vertices, we can compare lengths using the Pythagorean Theorem. The length of segment *AB* is $\sqrt{13}$ units because the segment is the hypotenuse of a right triangle with vertical side length 3 units and horizontal side length 2 units. The length of segment *DE* is $\sqrt{13}$ units as well, because this segment is also the hypotenuse of a right triangle with leg lengths 3 and 2 units.



The other sides of the triangles are congruent as well: The lengths of segments *BC* and *FE* are 1 unit each, and the lengths of segments *AC* and *DF* are each $\sqrt{10}$ units, because they are both hypotenuses of right triangles with leg lengths 1 and 3 units. So triangle *ABC* is congruent to triangle *DEF* by the Side-Side-Side Triangle Congruence Theorem.

Since triangle *ABC* is congruent to triangle *DEF*, there is a sequence of rigid motions that takes triangle *ABC* to triangle *DEF*. Here is one possible sequence: First, reflect triangle *ABC* across the *y*-axis. Then, translate the image by the directed line segment from (-1, 1) to (-3, 1).



NAME _____ DATE _____ PERIOD _____

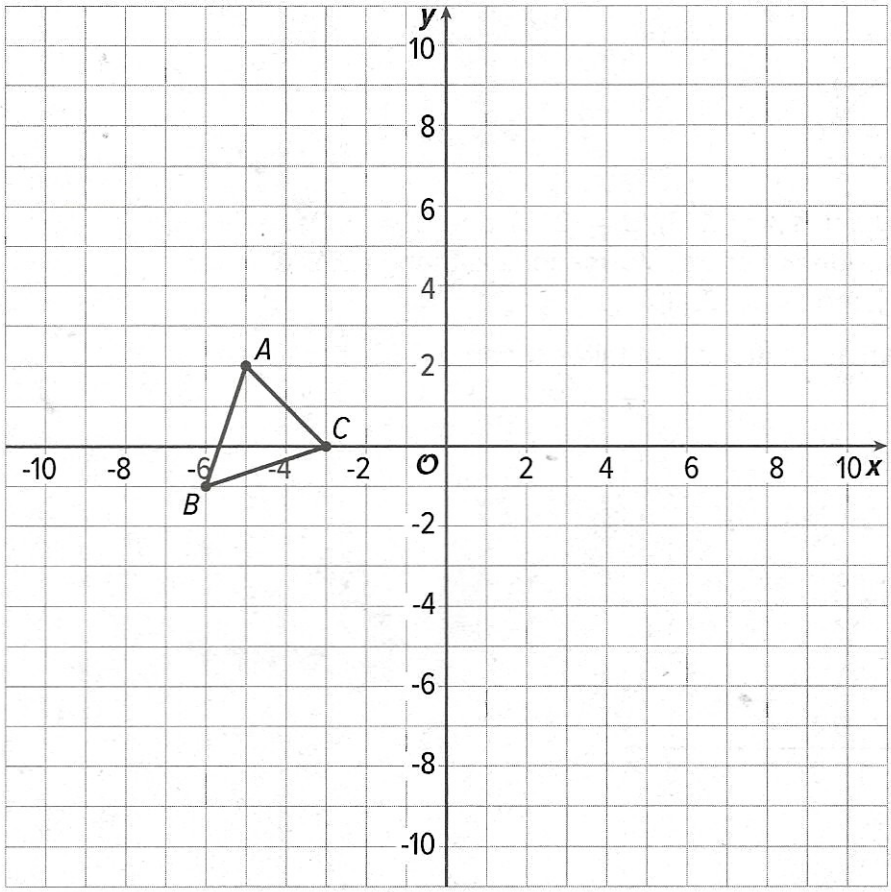
Practice

Rigid Transformations in the Plane

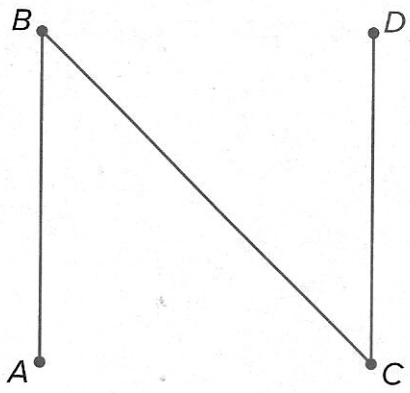
1. Reflect triangle *ABC* over the line $x = -3$.

   Translate the image by the directed line segment from (0, 0) to (4, 1).

   What are the coordinates of the vertices in the final image?

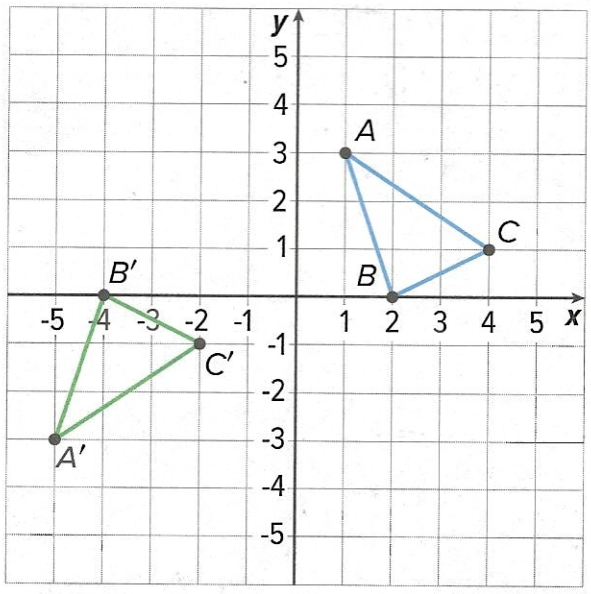


2. Three line segments form the letter N. Rotate the letter N counterclockwise around the midpoint of segment BC by 180 degrees. Describe the result. (Lesson 1-14)



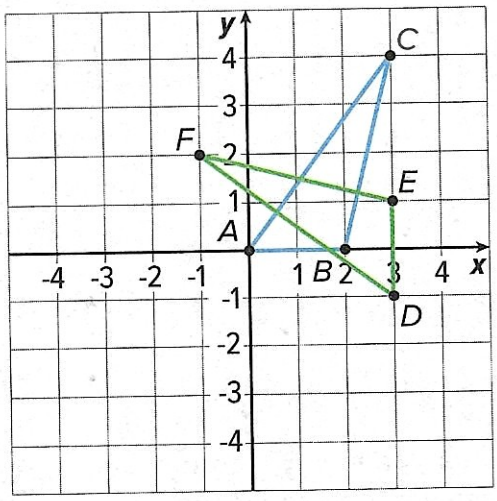
3. Triangle ABC has coordinates $A = (1, 3)$, $B = (2, 0)$, and $C = (4, 1)$. The image of this triangle after a sequence of transformations is triangle $A'B'C'$ where $A' = (-5, -3)$, $B' = (-4, 0)$, and $C' = (-2, -1)$.

Write a sequence of transformations that takes triangle ABC to triangle $A'B'C'$.



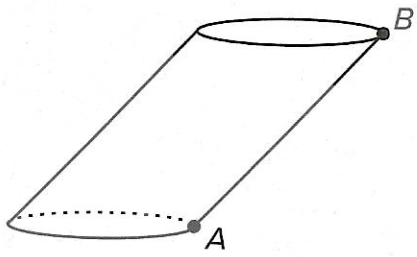
NAME _____ DATE _____ PERIOD _____

4. Prove triangle *ABC* is congruent to triangle *DEF*.

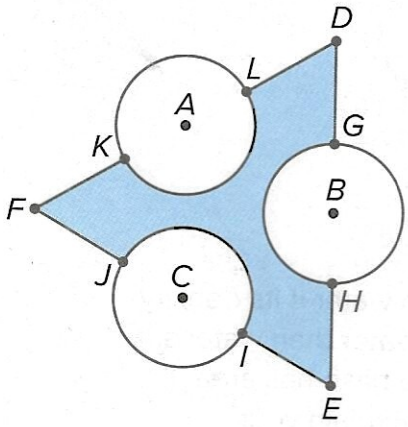


5. The density of water is 1 gram per cm^3. An object floats in water if its density is less than water's density, and it sinks if its density is greater than water's. Will a 1.17 gram diamond in the shape of a pyramid whose base has area 2 cm^2 and whose height is 0.5 centimeters sink or float? Explain your reasoning. **(Lesson 5-17)**

6. *Technology required.* An oblique cylinder with a base of radius 2 units is shown. The top of the cylinder can be obtained by translating the base by the directed line segment AB which has length 16 units. The segment AB forms a 30° angle with the plane of the base. What is the volume of the cylinder? (Lesson 5-11)



7. This design began from the construction of an equilateral triangle. Record at least 3 rigid transformations (rotation, reflection, translation) you see in the design. (Lesson 1-22)



Lesson 6-2

Transformations as Functions

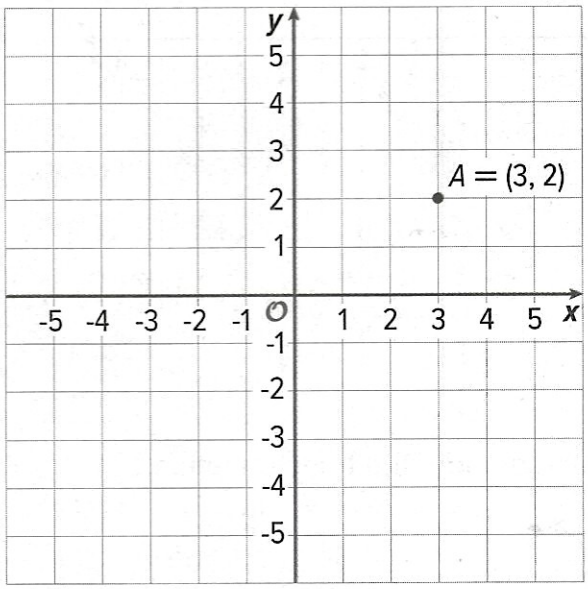
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's compare transformations to functions.

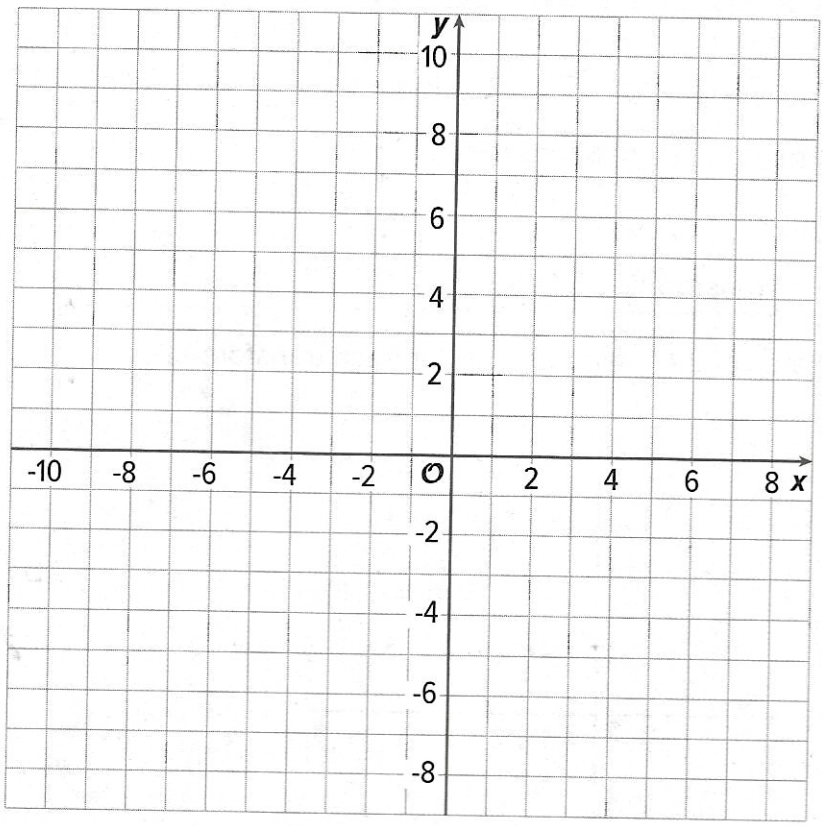
Warm Up

2.1 Math Talk: Transforming a Point

Mentally find the coordinates of the image of A under each transformation.



- Translate A by the directed line segment from (0, 0) to (0, 2).
- Translate A by the directed line segment from (0, 0) to (-4, 0).
- Reflect A across the x-axis.
- Rotate A 180 degrees clockwise using the origin as a center.



1. For each point (x, y), find its image under the transformation $(x + 12, y - 2)$.
   a. $A = (-10, 5)$
   b. $B = (-4, 9)$
   c. $C = (-2, 6)$

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2. Next, sketch triangle *ABC* and its image on the grid. What transformation is $(x, y) \rightarrow (x + 12, y - 2)$?

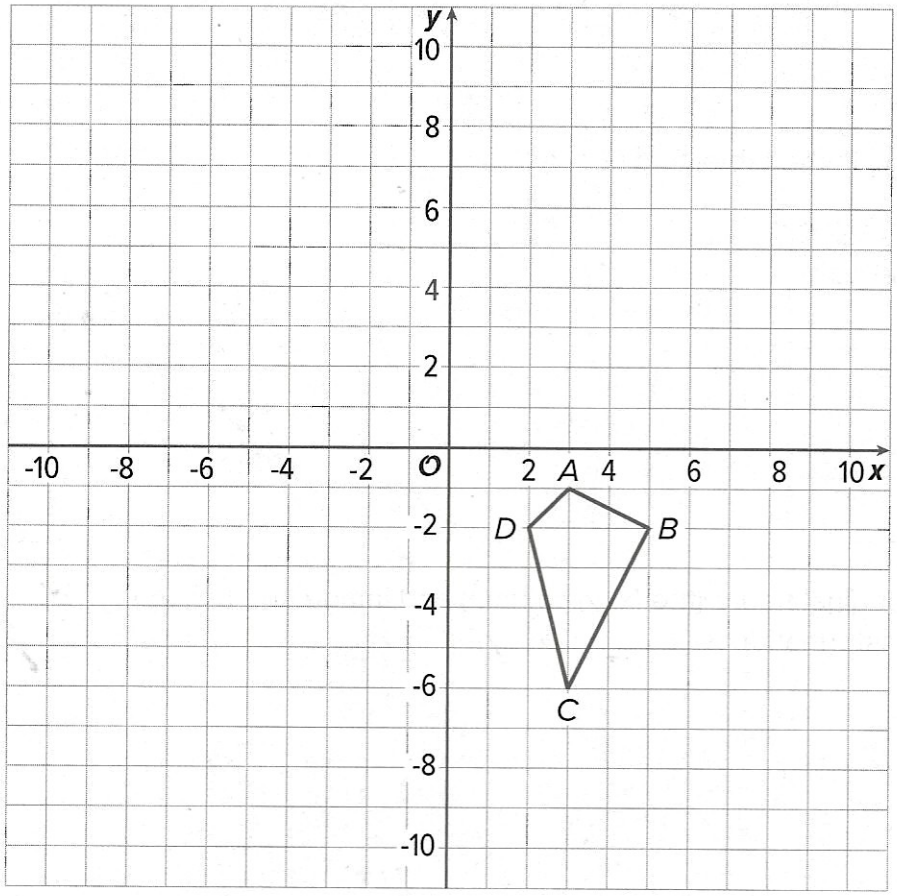
3. For each point (x, y) in the table, find $(2x, 2y)$.

(x, y)	$(2x, 2y)$
(-1, -3)	
(-1, 1)	
(5, 1)	
(5, -3)	

4. Next, sketch the original figure (the (x, y) column) and image (the $(2x, 2y)$ column). What transformation is $(x, y) \rightarrow (2x, 2y)$?

Activity

2.3 What Does it Do?



1. Here are some transformation rules. Apply each rule to quadrilateral *ABCD* and graph the resulting image. Then describe the transformation.

   a. Label this transformation *Q*: $(x, y) \rightarrow (2x, y)$

   b. Label this transformation *R*: $(x, y) \rightarrow (x, -y)$

   c. Label this transformation *S*: $(x, y) \rightarrow (y, -x)$

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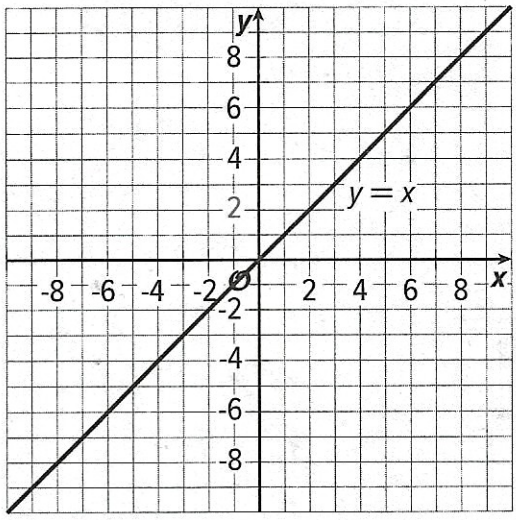
Are you ready for more?



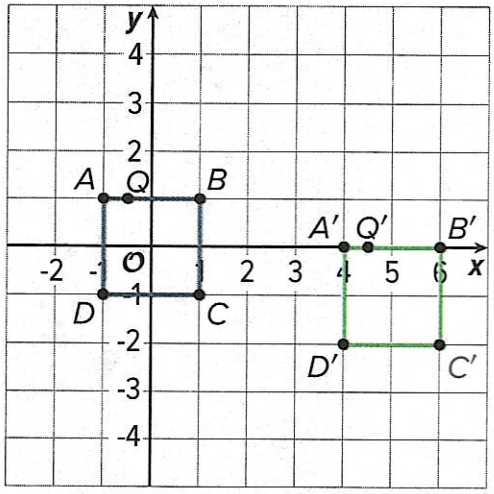
1. Plot the quadrilateral with vertices (4, -2), (8, 4), (8, -6), and (-6, -6). Label this quadrilateral A.
2. Plot the quadrilateral with vertices (-2, 4), (4, 8), (-6, 8), and (-6, -6). Label this quadrilateral A'.
3. How are the coordinates of quadrilateral A related to the coordinates of quadrilateral A'?

4. What single transformation takes quadrilateral A to quadrilateral A'?

Summary

Transformations as Functions

Square *ABCD* has been translated by the directed line segment from (-1, 1) to (4, 0). The result is square *A'B'C'D'*.



Here is a list of coordinates in the original figure and corresponding coordinates in the image. Do you see the rule for taking points in the original figure to points in the image?

Original Figure	Image
$A = (-1, 1)$	$A' = (4, 0)$
$B = (1, 1)$	$B' = (6, 0)$
$C = (1, -1)$	$C' = (6, -2)$
$D = (-1, -1)$	$D' = (4, -2)$
$Q = (-0.5, 1)$	$Q' = (4.5, 0)$

This table looks like a table that shows corresponding inputs and outputs of a function. A transformation is a special type of function that takes points in the plane as inputs and gives other points as outputs. In this case, the function's rule is to add 5 to the *x*-coordinate and subtract 1 from the *y*-coordinate.

We write the rule this way: $(x, y) \rightarrow (x + 5, y - 1)$.

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Practice

Transformations as Functions

1. Match each coordinate rule to a description of its resulting transformation.

   A. $(x, y) \rightarrow (x + 3, y)$

   B. $(x, y) \rightarrow (2x, 2y)$

   C. $(x, y) \rightarrow (x, y + 4)$

   D. $(x, y) \rightarrow (x, y - 4)$

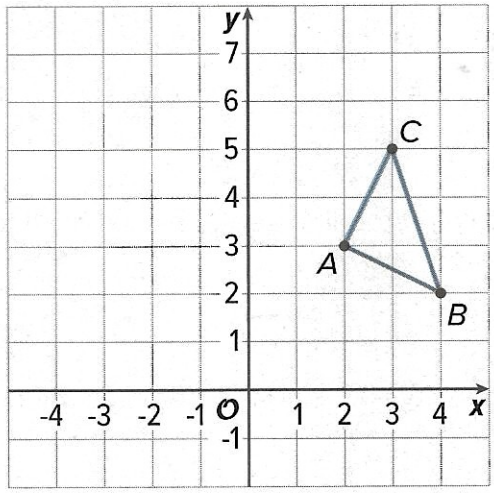
   E. $(x, y) \rightarrow (x - 3, y + 4)$

   1. Translate by the directed line segment from (0, 0) to (0, 4).
   2. Translate by the directed line segment from (0, 0) to (3, 0).
   3. Dilate using the origin as the center and a scale factor of 2.
   4. Translate by the directed line segment from (0, 0) to (0, -4).
   5. Translate by the directed line segment from (0, 0) to (-3, 4).

2. Respond to each question

   a. Draw the image of triangle ABC under the transformation $(x, y) \rightarrow (x - 4, y + 1)$. Label the result T.

   b. Draw the image of triangle ABC under the transformation $(x, y) \rightarrow (-x, y)$. Label the result R.



3. Here are some transformation rules. For each rule, describe whether the transformation is a rigid motion, a dilation, or neither.

   a. $(x, y) \rightarrow (x - 2, y - 3)$

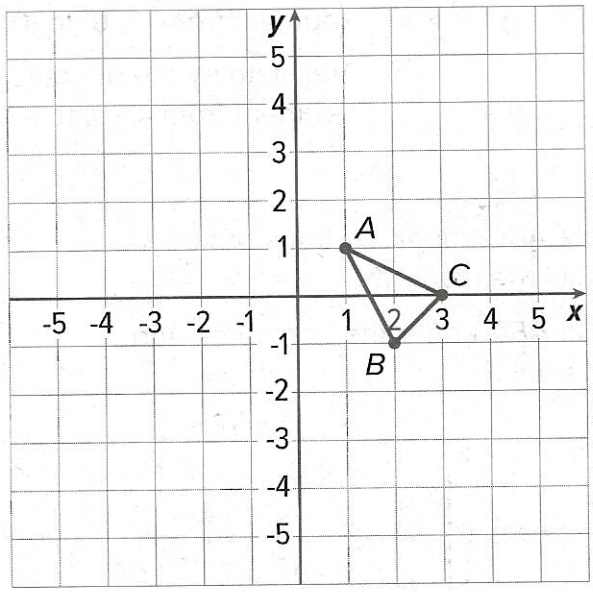
   b. $(x, y) \rightarrow (2x, 3y)$

   c. $(x, y) \rightarrow (3x, 3y)$

   d. $(x, y) \rightarrow (2 - x, y)$

4. Reflect triangle ABC over the line $x = 0$. Call this new triangle $A'B'C'$. Then reflect triangle $A'B'C'$ over the line $y = 0$. Call the resulting triangle $A''B''C''$.

   Which single transformation takes ABC to $A''B''C''$? (Lesson 6-1)



   (A.) Translate triangle ABC by the directed line segment from (1, 1) to (-2, 1).

   (B.) Reflect triangle ABC across the line $y = -x$.

   (C.) Rotate triangle ABC counterclockwise using the origin as the center by 180 degrees.

   (D.) Dilate triangle ABC using the origin as the center and a scale factor of 2.

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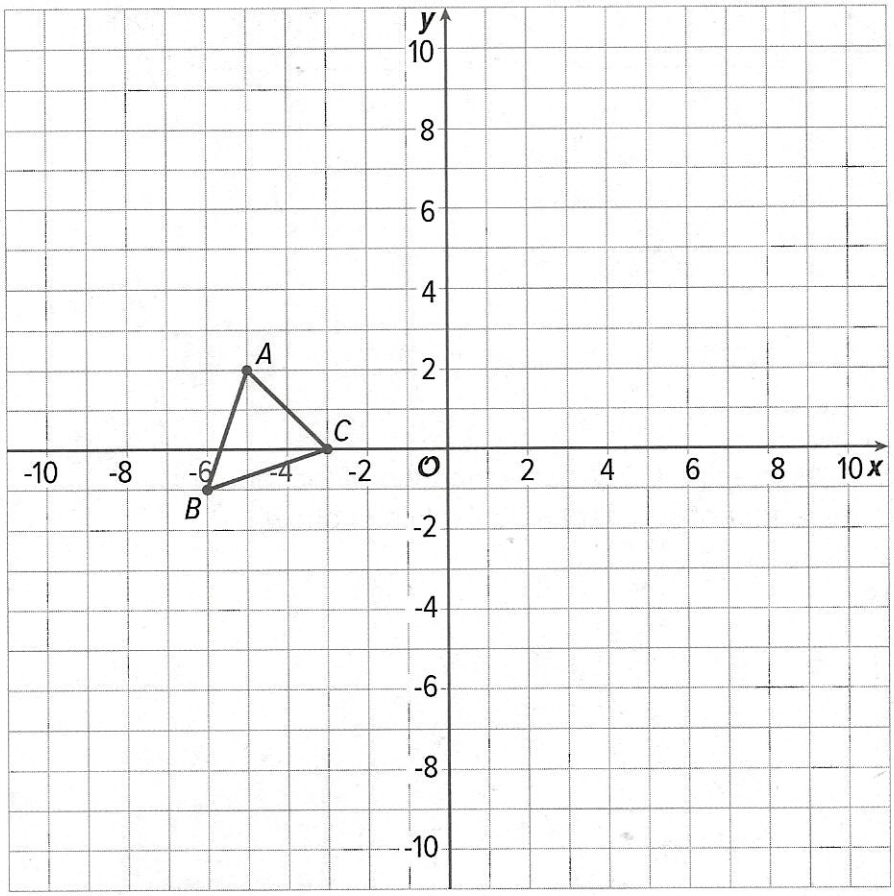
5. Reflect triangle ABC over the line $y = 2$.

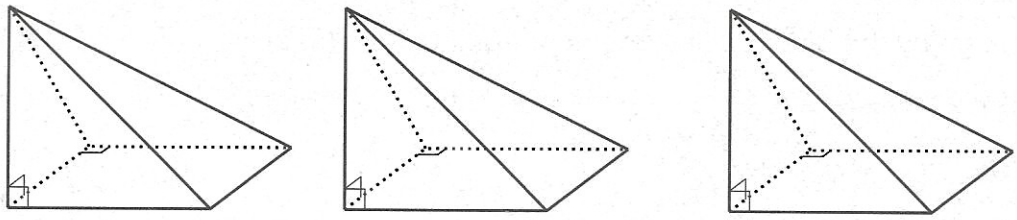
Translate the image by the directed line segment from (0, 0) to (3, 2).

What are the coordinates of the vertices in the final image? **(Lesson 6-1)**



6. The density of water is 1 gram per cm^3. An object floats in water if its density is less than water's density, and it sinks if its density is greater than water's. Will a cylindrical log with radius 0.4 meters, height 5 meters, and mass 1,950 kilograms sink or float? Explain your reasoning. **(Lesson 5-17)**

7. These 3 congruent square pyramids can be assembled into a cube with side length 3 feet. What is the volume of each pyramid? (Lesson 5-12)



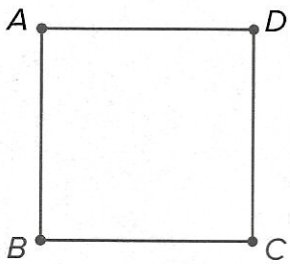
(A.) 1 cubic foot

(B.) 3 cubic feet

(C.) 9 cubic feet

(D.) 27 cubic feet

8. Reflect square *ABCD* across line *CD*. What is the ratio of the length of segment *AA'* to the length of segment *AD*? Explain or show your reasoning. (Lesson 2-1)



Lesson 6-3

Types of Transformations

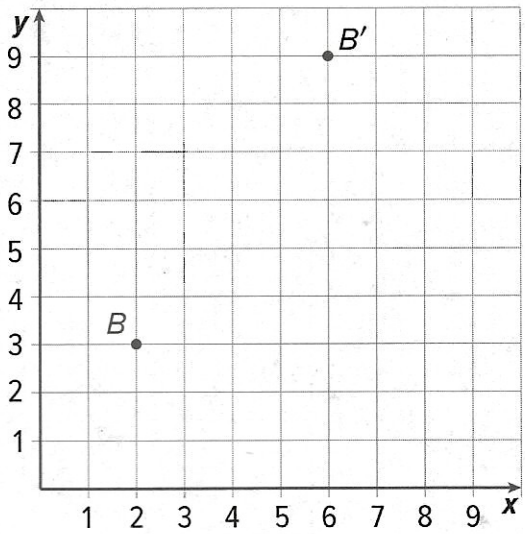
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's analyze transformations that produce congruent and similar figures.

Warm Up

3.1 Why is it a Dilation?

Point B was transformed using the coordinate rule $(x, y) \rightarrow (3x, 3y)$.



1. Add these auxiliary points and lines to create 2 right triangles: Label the origin P. Plot points $M = (2, 0)$ and $N = (6, 0)$. Draw segments PB', MB, and NB'.
2. How do triangles PMB and PNB' compare? How do you know?

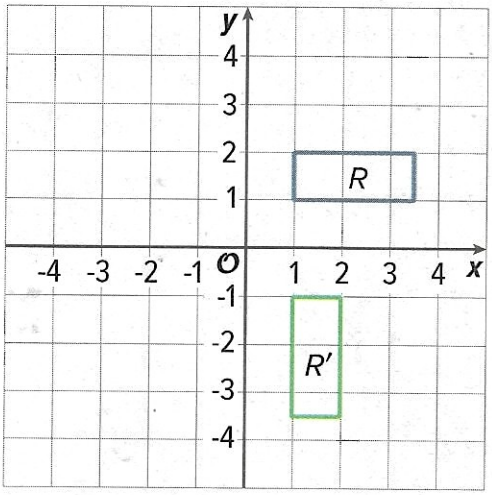
3. What must be true about the ratio $PB : PB'$?

Activity

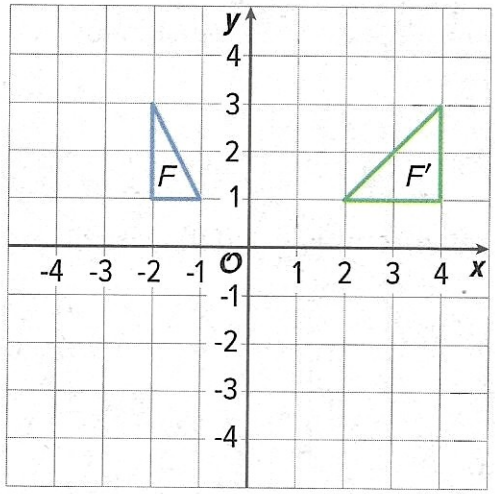
3.2 Congruent, Similar, Neither?

Match each image to its rule. Then, for each rule, decide whether it takes the original figure to a congruent figure, a similar figure, or neither. Explain or show your reasoning.

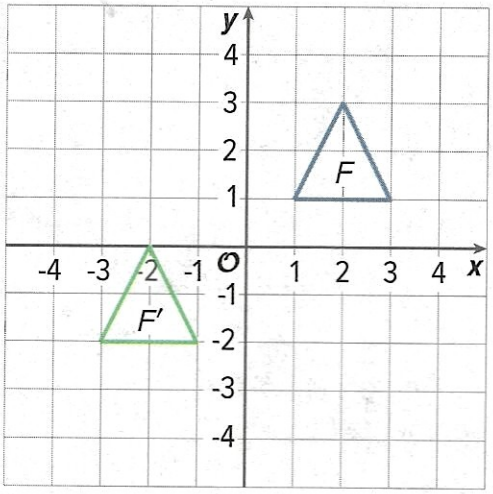
A



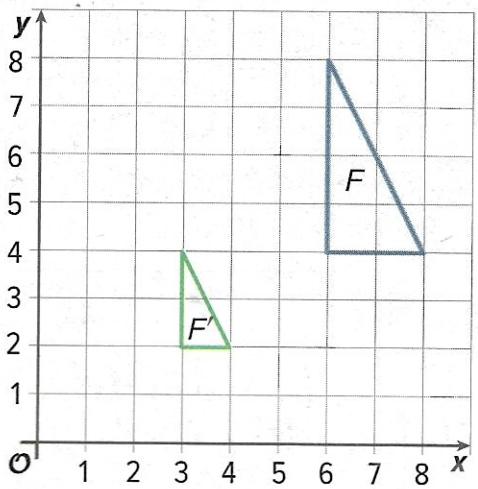
B



C



D



1. $(x, y) \rightarrow \left(\frac{x}{2}, \frac{y}{2}\right)$
2. $(x, y) \rightarrow (y, -x)$
3. $(x, y) \rightarrow (-2x, y)$
4. $(x, y) \rightarrow (x - 4, y - 3)$

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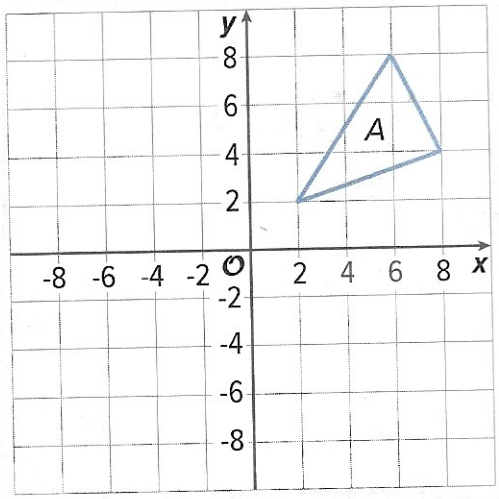
Are you ready for more?

Here is triangle A.

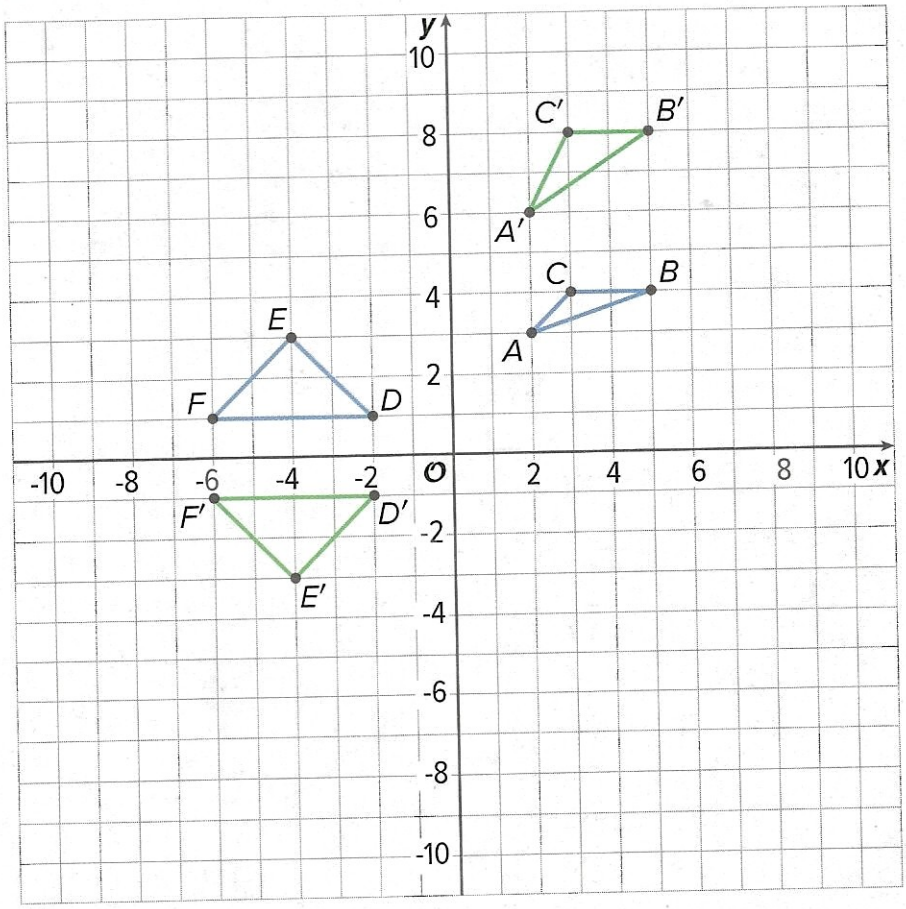


1. Reflect triangle A across the line $x = 2$.

2. Write a single rule that reflects triangle A across the line $x = 2$.

Activity

3.3 You Write the Rules



1. Write a rule that will transform triangle ABC to triangle $A'B'C'$.

2. Are ABC and $A'B'C'$ congruent? Similar? Neither? Explain how you know.

3. Write a rule that will transform triangle DEF to triangle $D'E'F'$.

4. Are DEF and $D'E'F'$ congruent? Similar? Neither? Explain how you know.

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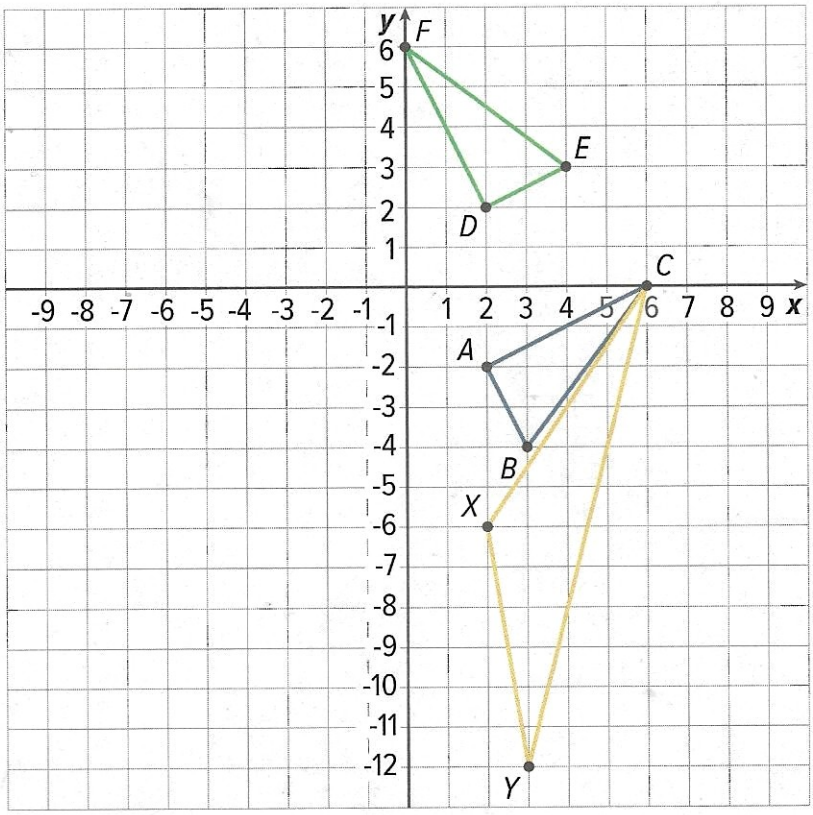
Summary

Types of Transformations

Triangle *ABC* has been transformed in two different ways:

- $(x, y) \rightarrow (-y, x)$, resulting in triangle *DEF*
- $(x, y) \rightarrow (x, 3y)$, resulting in triangle *XYC*



Let's analyze the effects of the first transformation. If we calculate the lengths of all the sides, we find that segments *AB* and *DE* each measure $\sqrt{5}$ units, *BC* and *EF* each measure 5 units, and *AC* and *DF* each measure $\sqrt{20}$ units. The triangles are congruent by the Side-Side-Side Triangle Congruence Theorem. That is, this transformation leaves the lengths and angles in the triangle the same—it is a rigid transformation.

Not all transformations keep lengths or angles the same. Compare triangles *ABC* and *XYC*. Angle *X* is larger than angle *A*. All of the side lengths of *XYC* are larger than their corresponding sides. The transformation $(x, y) \rightarrow (x, 3y)$ stretches the points on the triangle 3 times farther away from the *x*-axis. This is not a rigid transformation. It is also not a dilation since the corresponding angles are not congruent.

Practice

Types of Transformations

1. Complete the table and determine the rule for this transformation.

Input	Output
(2, -3)	(-3, 2)
(4, 5)	(5, 4)
(4)	(4, 0)
(1, 6)	
	(-1, -2)
(x, y)	

2. Write a rule that describes this transformation.

Original Figure	Image
(5, 1)	(2, -1)
(-3, 4)	(-6, -4)
(1, -2)	(-2, 2)
(-1, -4)	(-4, 4)

3. Select all the transformations that produce congruent images.

A. dilation

B. horizontal stretch

C. reflection

D. rotation

E. translation

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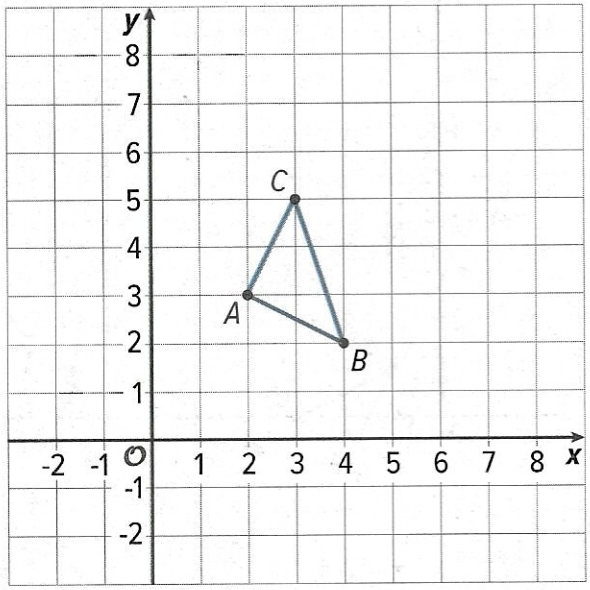
4. Here are some transformation rules. For each transformation, first predict what the image of triangle *ABC* will look like. Then compute the coordinates of the image and draw it. (Lesson 6-2)



a. $(x, y) \rightarrow (x - 4, y - 1)$

b. $(x, y) \rightarrow (y, x)$

c. $(x, y) \rightarrow (1.5x, 1.5y)$

5. A cylinder has radius 3 inches and height 5 inches. A cone has the same radius and height. (Lesson 5-13)

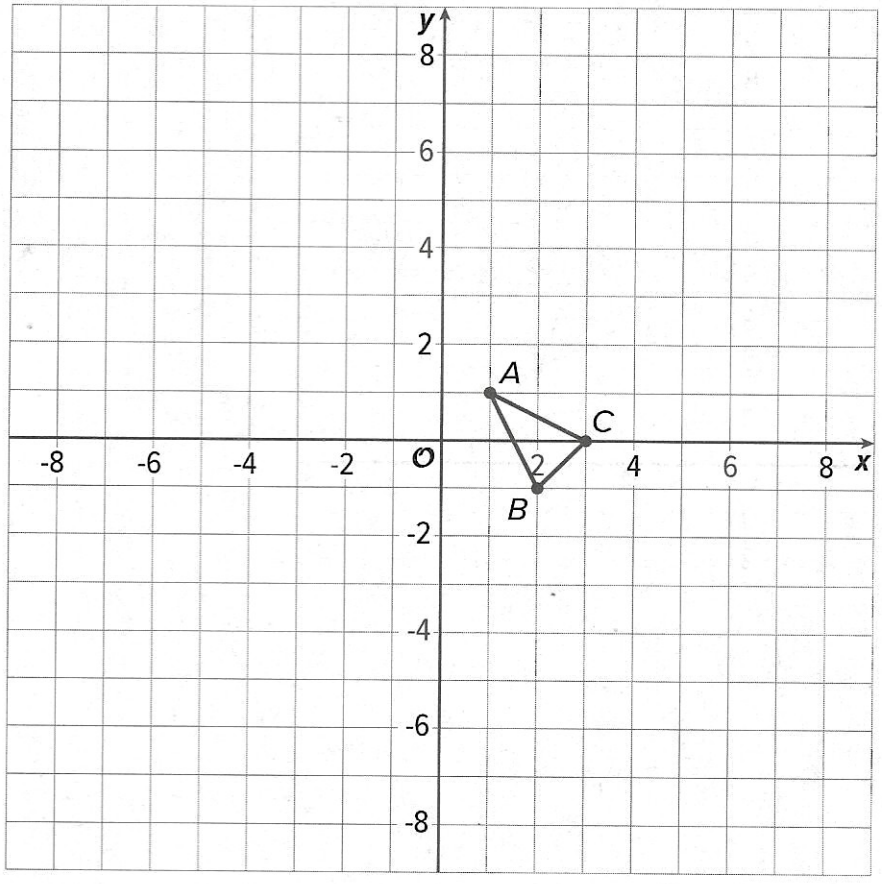
a. Find the volume of the cylinder.

b. Find the volume of the cone.

c. What fraction of the cylinder's volume is the cone's volume?

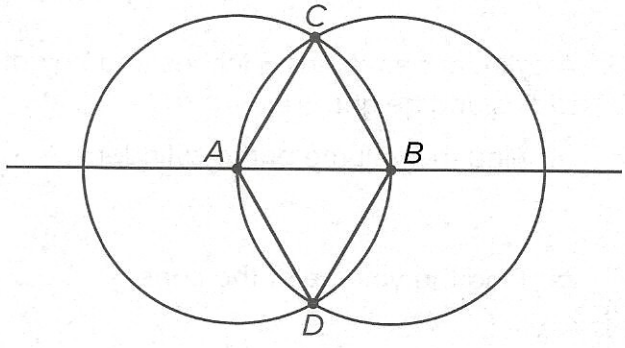
6. Reflect triangle ABC over the line $x = -2$. Call this new triangle $A'B'C'$. Then reflect triangle $A'B'C'$ over the line $x = 0$. Call the resulting triangle $A''B''C''$.

Describe a single transformation that takes ABC to $A''B''C''$. **(Lesson 6-1)**



7. In the construction, A is the center of one circle, and B is the center of the other. **(Lesson 1-2)**

Explain why segments AC, BC, AD, BD, and AB have the same length.



Lesson 6-4

Distances and Circles

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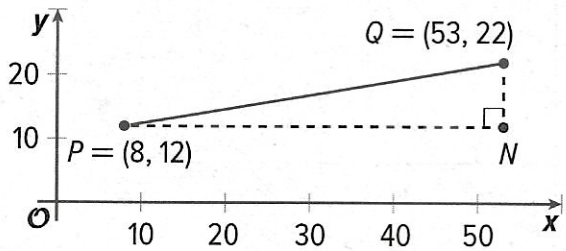
Learning Goal Let's build an equation for a circle.

Warm Up

4.1 Going the Distance



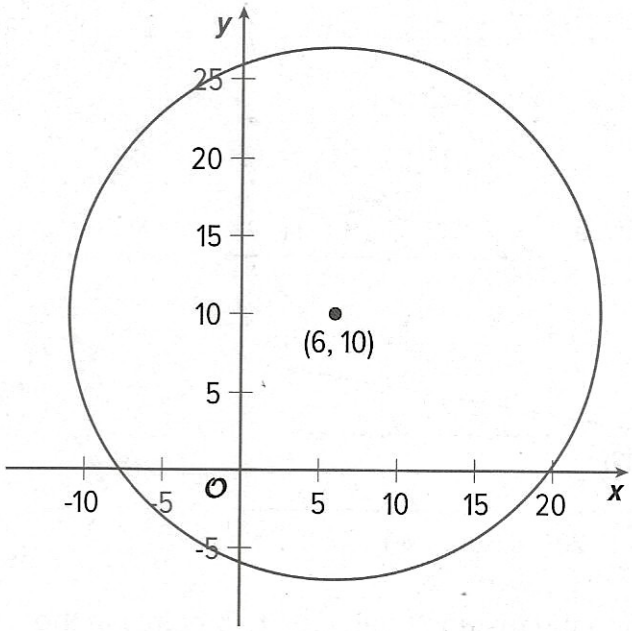
Andre says, "I know that I can find the distance between two points in the plane by drawing in a right triangle and using the Pythagorean Theorem. But I'm not sure how to find the lengths of the legs of the triangle when I can't just count the squares on the graph."

Explain to Andre how he can find the lengths of the legs in the triangle in the image. Then, calculate the distance between points P and Q.

Activity

4.2 Circling the Problem

The image shows a circle with center (6, 10) and radius 17 units.



1. The point (14, 25) looks like it might be on the circle. Verify if it really is on the circle. Explain or show your reasoning.

2. The point (22, 3) looks like it might be on the circle. Verify if it really is on the circle. Explain or show your reasoning.

3. In general, how can you check if a particular point (x, y) is on the circle?

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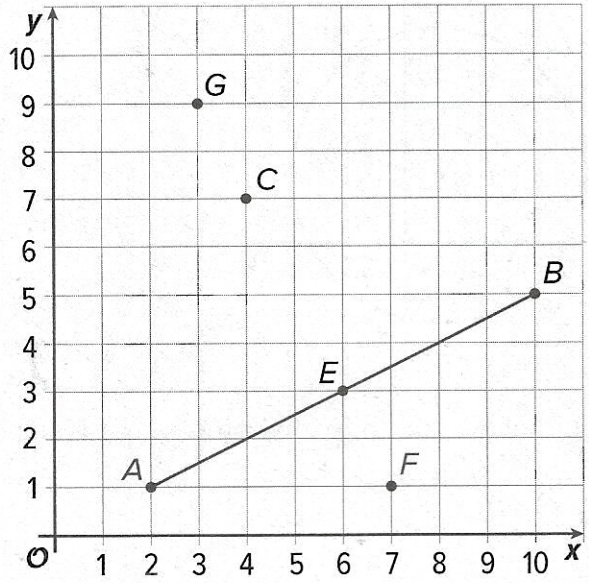
Are you ready for more?

The image shows segment AB and several points.



1. Calculate the distance from each point to the endpoints of segment AB.

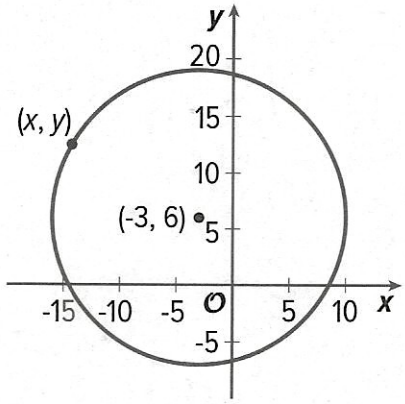
	Point A	Point B
Point G		
Point C		
Point E		
Point F		

2. What do the distances tell you about points G, E, C, and F?

Activity

4.3 Building an Equation for a Circle

The image shows a circle with center (-3, 6) and radius 13 units.



1. Write an equation that would allow you to test whether a particular point (x, y) is on the circle.

2. Use your equation to test whether (9, 1) is on the circle.

3. Suppose you have a circle with center (h, k) and radius r. Write an equation that would allow you to test whether a particular point (x, y) is on the circle.

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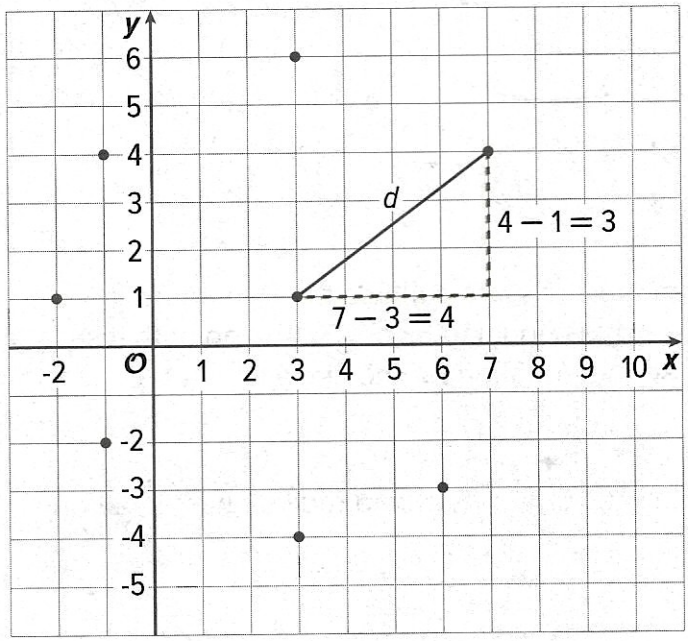
Summary

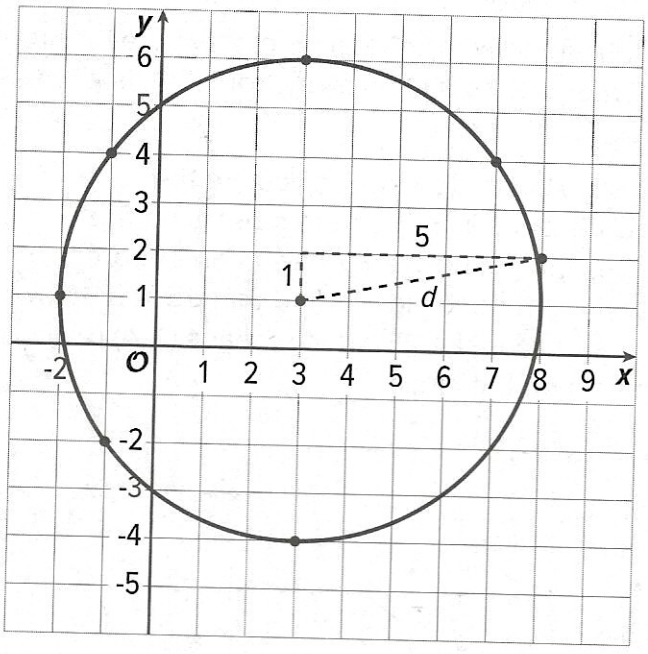
Distances and Circles

The diagram shows the point (3, 1), along with several points that are 5 units away from (3, 1). The set of *all* points 5 units away from (3, 1) is a circle with center (3, 1) and radius 5.

The point (7, 4) appears to be on this circle. To verify, calculate the distance from (7, 4) to (3, 1). If this distance is 5, then the point is on the circle. Let d stand for the distance, and set up the Pythagorean Theorem: $(7-3)^2+(4-1)^2=d^2$. Evaluate the left hand side to find that $25=d^2$. Now d is the positive number that squares to make 25, which means (7, 4) really is 5 units away from (3, 1). This point is on the circle.



The point (8, 2) also looks like it could be on the circle. To find its distance from (3, 1), we can do a similar calculation: $(8-3)^2+(2-1)^2=d^2$. Evaluating the left side, we get $26=d^2$. This means that d must be a little more than 5. So (8, 2) does not lie on the circle.



To check if any point (x, y) is on the circle, we can use the Pythagorean Theorem to see if $(x-3)^2+(y-1)^2$ is equal to 5^2 or 25. Any point that satisfies this condition is on the circle, so the equation for the circle is $(x-3)^2+(y-1)^2=25$.

By the same reasoning, a circle with center (h, k) and radius r has equation $(x-h)^2+(y-k)^2=r^2$.

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Practice

Distances and Circles

1. Match each equation to its description.

A. circle centered at (0, -4) with a radius of 3

B. circle centered at (1, -4) with a radius of $\sqrt{3}$

C. circle centered at (1, 4) with a radius of $\sqrt{3}$

D. circle centered at (1, 0) with a radius of 3

E. circle centered at (1, -4) with a radius of 3

1. $(x-1)^2 + y^2 = 9$
2. $x^2 + (y+4)^2 = 9$
3. $(x-1)^2 + (y-4)^2 = 3$
4. $(x-1)^2 + (y+4)^2 = 9$
5. $(x-1)^2 + (y+4)^2 = 3$

2. Write an equation of a circle that is centered at (-3, 2) with a radius of 5.

(A.) $(x-3)^2 + (y+2)^2 = 5$

(B.) $(x+3)^2 + (y-2)^2 = 5$

(C.) $(x-3)^2 + (y+2)^2 = 25$

(D.) $(x+3)^2 + (y-2)^2 = 25$

3. Respond to each question.

   a. Plot the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$ on the same coordinate plane.

   b. Find the image of any point on $x^2 + y^2 = 4$ under the transformation $(x, y) \rightarrow \left(\frac{1}{2}x, \frac{1}{2}y\right)$.

   c. What do you notice about $x^2 + y^2 = 4$ and $x^2 + y^2 = 1$?

4. $(x, y) \rightarrow (x - 3, 4 - y)$ is an example of a transformation called a glide reflection. Complete the table using the rule.

   Does this glide reflection produce a triangle congruent to the original? **(Lesson 6-3)**

Input	Output
(1, 1)	(-2, 3)
(6, 1)	
(3, 5)	

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5. The triangle whose vertices are (1, 1), (5, 3), and (4, 5) is transformed by the rule $(x, y) \rightarrow (3x, 3y)$. Is the image similar or congruent to the original figure? **(Lesson 6-3)**

(A.) The image is congruent to the original triangle.

(B.) The image is similar but not congruent to the original triangle.

(C.) The image is neither similar nor congruent to the original triangle.

6. Match each coordinate rule to a description of its resulting transformation. **(Lesson 6-2)**

A. $(x, y) \rightarrow (3x, 3y)$

B. $(x, y) \rightarrow (x - 3, y - 3)$

C. $(x, y) \rightarrow (x + 3, y + 3)$

D. $(x, y) \rightarrow (x - 3, y)$

E. $(x, y) \rightarrow (x + 3, y)$

F. $(x, y) \rightarrow (x, y - 3)$

G. $(x, y) \rightarrow (x, y + 3)$

1. Translate along the directed line segment from (0, 0) to (-3, 0).

2. Translate along the directed line segment from (0, 0) to (0, -3).

3. Translate along the directed line segment from (0, 0) to (3, 0).

4. Translate along the directed line segment from (0, 0) to (0, 3).

5. Translate along the directed line segment from (0, 0) to (3, 3).

6. Translate along the directed line segment from (0, 0) to (-3, -3).

7. Dilate using the origin as the center and a scale factor of 3.

7. A cone-shaped container is oriented with its circular base on the top and its apex at the bottom. It has a radius of 18 inches and a height of 6 inches. The cone starts filling up with water. What fraction of the volume of the cone is filled when the water reaches a height of 2 inches? **(Lesson 5-14)**

(A) $\frac{1}{729}$

(B) $\frac{1}{27}$

(C) $\frac{1}{9}$

(D) $\frac{1}{3}$

Lesson 6-5

Squares and Circles

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's see how the distributive property can relate to equations of circles.

Warm Up

5.1 Math Talk: Distribution

Distribute each expression mentally.

$5(x + 3)$

$x(x - 3)$

$(x + 4)(x + 2)$

$(x - 5)(x - 5)$

Activity

5.2 Perfectly Square

1. Apply the distributive property to each expression.
   a. $(x - 7)(x - 7)$
   b. $(x + 4)^2$
   c. $(x - 10)^2$
   d. $(x + 1)^2$
2. Look at your results. Each of these expressions is called a *perfect square trinomial*. Why?

3. Which of these expressions are perfect square trinomials? If you get stuck, look for patterns in your earlier work.
   a. $x^2 - 6x + 9$
   b. $x^2 + 10x + 20$
   c. $x^2 + 18x + 81$
   d. $x^2 - 2x + 1$
   e. $x^2 + 4x + 16$
4. Rewrite the perfect square trinomials you identified as squared binomials.

Activity

5.3 Back and Forth

1. Here is the equation of a circle: $(x - 2)^2 + (y + 7)^2 = 10^2$
   a. What are the center and radius of the circle?
   b. Apply the distributive property to the squared binomials and rearrange the equation so that one side is 0. This is the form in which many circle equations are written.
2. This equation looks different, but also represents a circle:
   $x^2 + 6x + 9 + y^2 - 10y + 25 = 64$
   a. How can you rewrite this equation to find the center and radius of the circle?
   b. What are the center and radius of the circle?

Are you ready for more?

In three-dimensional space, there are 3 coordinate axes, called the x-axis, the y-axis, and the z-axis. Write an equation for a sphere with center (a, b, c) and radius r.

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Summary

Squares and Circles

Suppose we square several binomials, or expressions that contain 2 terms. We get trinomials, or expressions that contain 3 terms. Does any pattern emerge in the results?

$(x + 6)^2 = x^2 + 12x + 36$

$(x - 8)^2 = x^2 - 16x + 64$

$(x + 5)^2 = x^2 + 10x + 25$

Each of the expressions on the right are called perfect square trinomials because they are the result of multiplying an expression by itself. There is a pattern in the results: When the coefficient of x^2 in a trinomial is 1, if the constant term is the square of half the coefficient of x, then the expression is a perfect square trinomial.

For example, $x^2 - 14x + 49$ is a perfect square trinomial because the constant term, 49, can be rewritten as $(-7)^2$, and half of -14 is -7. This expression can be rewritten as a squared binomial: $(x - 7)^2$.

Two squared binomials show up in the equation for circles: $(x - h)^2 + (y - k)^2 = r^2$. Equations for circles are sometimes written in different forms, but we can rearrange them to help find the center and radius of the circle. For example, suppose the equation of a circle is written like this:

$x^2 - 22x + 121 + y^2 + 2y + 1 = 225$

We can't immediately identify the center and radius of the circle. However, if we rewrite the two perfect square trinomials as squared binomials and rewrite the right side in the form r^2, the center and radius will be easier to recognize.

The first 3 terms on the left side, $x^2 - 22x + 121$, can be rewritten as $(x - 11)^2$. The remaining terms, $y^2 + 2y + 1$, can be rewritten as $(y + 1)^2$. The right side, 225, can be rewritten as 15^2. Let's put it all together.

$(x - 11)^2 + (y + 1)^2 = 15^2$

Now we can see that the center of the circle is (11, -1) and the circle's radius measures 15 units.

Practice

Squares and Circles

1. Match each quadratic expression with an equivalent expression in factored form.

A. $x^2 + 6x$	1. $(x + 7)(x - 1)$
B. $x^2 + 6x + 5$	2. $(x + 5)(x + 1)$
C. $x^2 + 6x - 7$	3. $(x + 4)(x + 2)$
D. $x^2 + 6x + 8$	4. $(x + 3)(x + 3)$
E. $x^2 + 6x + 9$	5. $x(x + 6)$

2. An equation of a circle is $x^2 - 8x + 16 + y^2 + 10y + 25 = 81$.
   a. What is the radius of the circle?
   b. What is the center of the circle?

3. Write 3 perfect square trinomials. Then rewrite them as squared binomials.

   a.

   b.

   c.

4. Write an equation of the circle that has a diameter with endpoints (12, 3) and (-18, 3). **(Lesson 6-4)**

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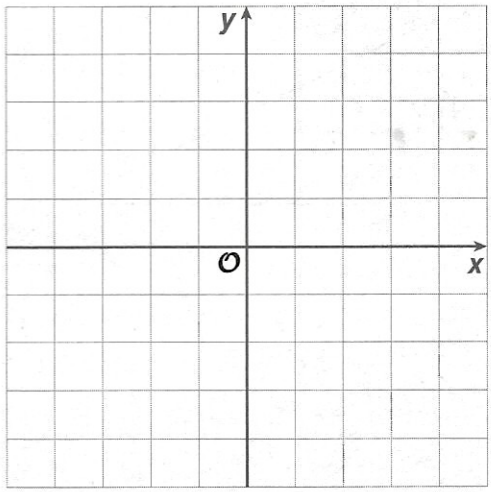
5. Respond to each question.

   a. Graph the circle $(x - 2)^2 + (y - 1)^2 = 25$.



   b. For each point, determine if it is on the circle. If not, decide whether it is inside the circle or outside of the circle.

      i. (4, 0)

      ii. (-3, 3)

      iii. (-2, -2)

   c. How can you use distance calculations to decide if a point is inside, on, or outside a circle? (Lesson 6-4)

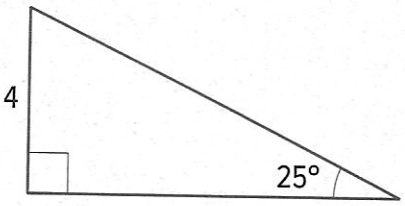
6. The triangle whose vertices are (2, 5), (3, 1), and (4, 2) is transformed by the rule $(x, y) \rightarrow (x - 2, y + 4)$. Is the image similar or congruent to the original figure? (Lesson 6-3)

   (A.) The image is congruent to the original triangle.

   (B.) The image is similar but not congruent to the original triangle.

   (C.) The image is neither similar nor congruent to the original triangle.

7. *Technology required.* A triangular prism has height 6 units. The base of the prism is shown in the image. What is the volume of the prism? Round your answer to the nearest tenth. **(Lesson 5-15)**



Lesson 6-6

Completing the Square

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's rewrite equations to find the center and radius of a circle.

Warm Up

6.1 Fill in the Box

For each expression, what value would need to be in the box in order for the expression to be a perfect square trinomial?

1. $x^2 + 10x + \square$
2. $x^2 - 16x + \square$
3. $x^2 + 40x + \square$
4. $x^2 + 5x + \square$

Activity

6.2 Complete the Process

Here is the equation of a circle: $x^2 + y^2 - 6x - 20y + 105 = 0$

Elena wants to find the center and radius of the circle. Here is what she's done so far.

Step 1: $x^2 - 6x + y^2 - 20y = -105$

Step 2: $x^2 - 6x + 9 + y^2 - 20y + 100 = -105 + 9 + 100$

Step 3: $x^2 - 6x + 9 + y^2 - 20y + 100 = 4$

1. What did Elena do in the first step?

2. Why did Elena add 9 and 100 to the *left* side of the equation in Step 2?

3. Why did Elena add 9 and 100 to the *right* side of the equation in Step 2?

4. What should Elena do next?

5. What are the center and radius of this circle?

6. Draw a graph of the circle.

Activity
6.3 Your Turn

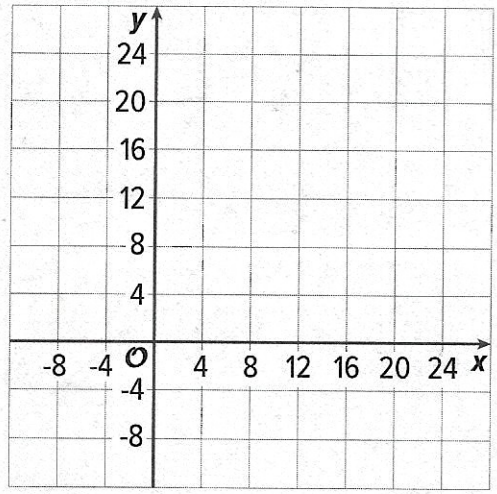
Here is the equation of a circle: $x^2 + y^2 - 2x + 4y - 4 = 0$

1. Find the center and radius of the circle. Explain or show your reasoning.

2. Draw a graph of the circle.

Are you ready for more?

Triangulation is a process of using 3 distances from known landmarks to locate an exact position. Find a point that is located 5 units from the point (3, 4), 13 units from the point (11, -4), and 17 units from the point (21, 16). Use the coordinate grid if it's helpful.



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Summary

Completing the Square

Here is an equation for a circle: $x^2 + y^2 - 4x + 6y - 3 = 0$. If we want to find the center and radius of the circle, we can rewrite the equation in the form $(x - h)^2 + (y - k)^2 = r^2$.

Start by rearranging the terms in the equation to make it easier to work with. Group terms that include the same variable and move the -3 to the right side of the equation.

$x^2 - 4x + y^2 + 6y = 3$

We want the left side to include 2 perfect square trinomials—then, those trinomials can be rewritten in factored form to get the equation in the form we need. To create perfect square trinomials, we can add values to the left side. We'll keep the equation balanced by adding those same values to the other side.

$x^2 - 4x + \square + y^2 + 6y + \square = 3 + \square + \square$

For the expression $x^2 - 4x$, we need to add 4 to get a perfect square trinomial. For the expression $y^2 + 6y$, we need to add 9. Add these values to *both* sides of the equation. Then, combine the numbers on the right side.

$x^2 - 4x + 4 + y^2 + 6y + 9 = 3 + 4 + 9$

$x^2 - 4x + 4 + y^2 + 6y + 9 = 16$

Now rewrite the perfect square trinomials as squared binomials, and write the 16 in the form r^2.

$(x - 2)^2 + (y + 3)^2 = 4^2$

The circle has center (2, -3) and radius 4 units.

Practice

Completing the Square

1. Suppose a classmate missed the lessons on completing the square to find the center and radius of a circle. Explain the process to them. If it helps, use a problem you've already done as an example.

2. Match each expression with the value needed in the box in order for the expression to be a perfect square trinomial.

A. $x^2 - 8x + \square$	1. 16
B. $x^2 + 20x + \square$	2. 20.25
C. $x^2 - 16x + \square$	3. 64
D. $x^2 + 9x + \square$	4. 100

3. Find the center and radius of the circle represented by the equation $x^2 + y^2 + 4x - 10y + 20 = 0$.

4. Select **all** the expressions that can be factored into a squared binomial. (Lesson 6-5)

(A.) $y^2 + 2y + 1$	(D.) $x^2 - 10x + 25$
(B.) $w^2 + 5w + \frac{25}{4}$	(E.) $x^2 + 10x + 25$
(C.) $y^2 - 10y + 5$	(F.) $w^2 + 20w + 40$

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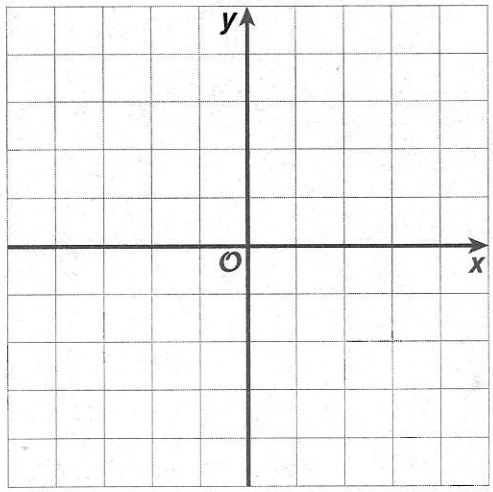
5. An equation of a circle is given by $(x + 3)^2 + (y - 9)^2 = 5^2$. Apply the distributive property to the squared binomials and rearrange the equation so that one side is 0. (Lesson 6-5)

6. Respond to each question.

   a. Graph the circle $(x + 1)^2 + (y - 3)^2 = 16$.



   b. Find the distance from the center of the circle to each point on the list.

      i. (2, 1)

      ii. (4, 1)

      iii. (3, 3)

   c. What do these distances tell you about whether each point is inside, on, or outside the circle? (Lesson 6-4)

7. The triangle whose vertices are (3, -1), (2, 4), and (5, 1) is transformed by the rule $(x, y) \rightarrow (2x, 5y)$. Is the image similar or congruent to the original figure? (Lesson 6-3)

   (A.) The image is congruent to the original triangle.

   (B.) The image is similar but not congruent to the original triangle.

   (C.) The image is neither similar nor congruent to the original triangle.

8. A cube has side length 3 inches. A sphere has a radius of 3 inches. (Lesson 5-16)

   a. Before doing any calculations, predict which solid has greater surface area to volume ratio.

   b. Calculate the surface area, volume, and surface area to volume ratio for each solid.

Lesson 6-7

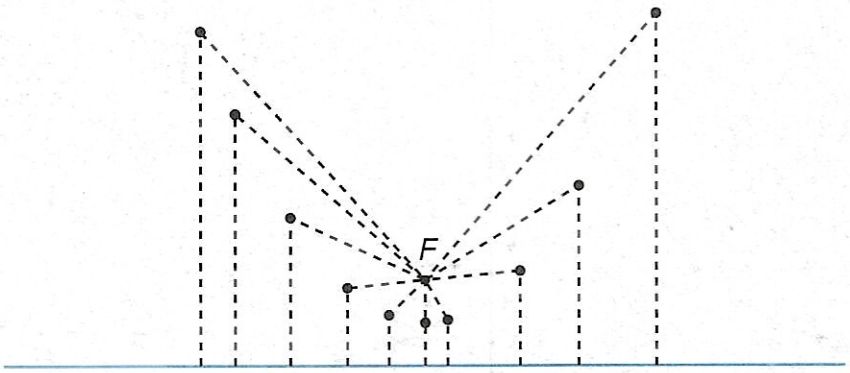
Distances and Parabolas

NAME _____ DATE _____ PERIOD _____

Learning Goal Let's analyze the set of points that are the same distance from a given point and a given line.

Warm Up

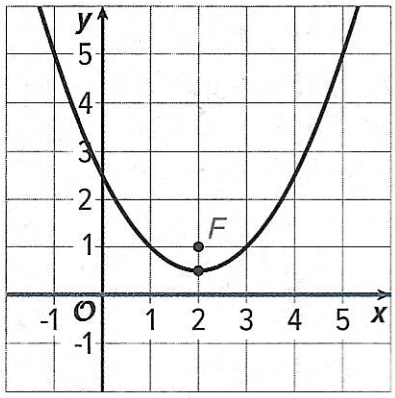
7.1 Notice and Wonder: Distances

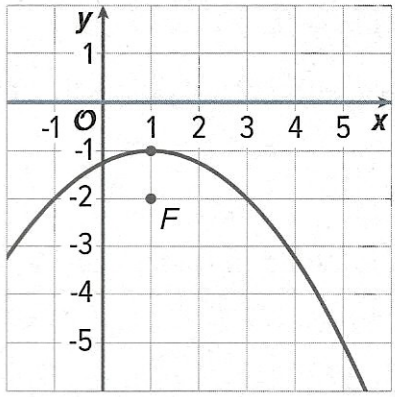


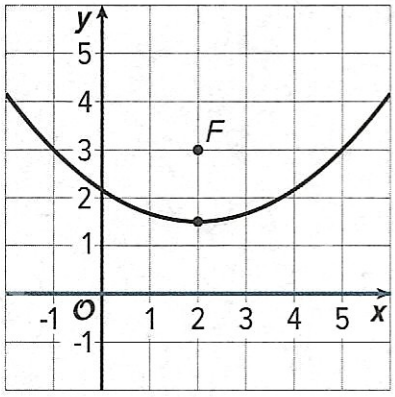
What do you notice? What do you wonder?

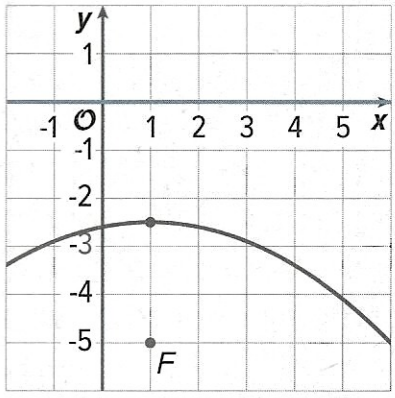
Activity
7.2 Into Focus

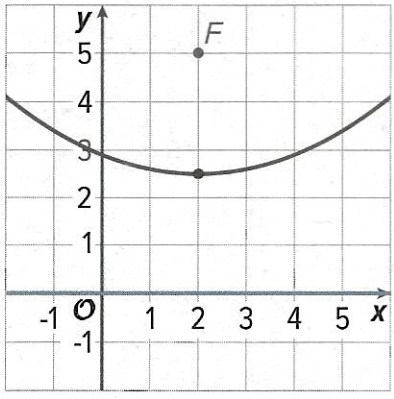
Here are several images of **parabolas**.

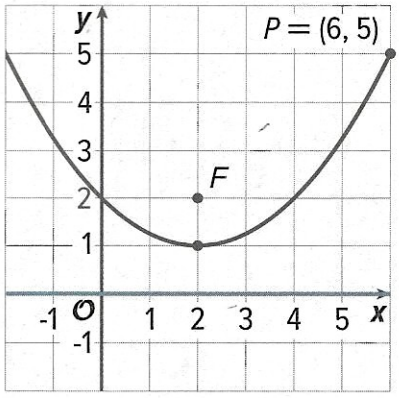












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Look at the **focus** and **directrix** of each parabola. In each case, the directrix is the *x*-axis.

1. How does the distance between the focus and the directrix affect the shape of the parabola?

2. What seems to need to be true in order for the parabola to open downward (that is, to be shaped like a hill instead of a valley)?

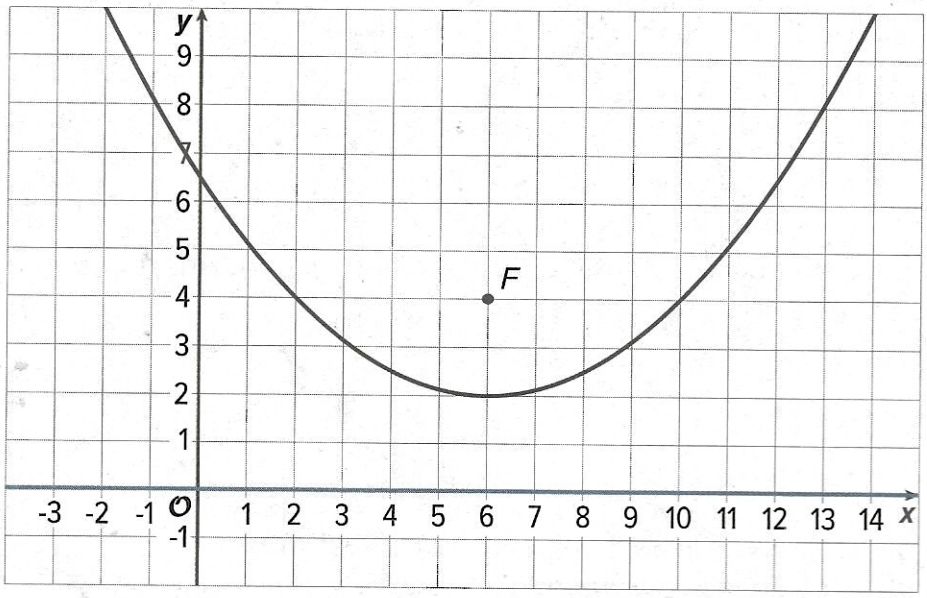
3. The vertex of the parabola is the lowest point on the curve if it opens upward, or the highest if it opens downward. Where is the vertex located in relationship to the focus and the directrix?

4. In the final image, the directrix is on the *x*-axis and the focus is the point (2, 2). Point *P* on the parabola is plotted.
   a. What is the distance between point *P* and the directrix?
   b. What does this tell you about the distance between *P* and *F*?

Activity

7.3 On Point

The image shows a parabola with focus (6, 4) and directrix $y = 0$ (the x-axis).



1. The point (11, 5) looks like it might be on the parabola. Determine if it really is on the parabola. Explain or show your reasoning.

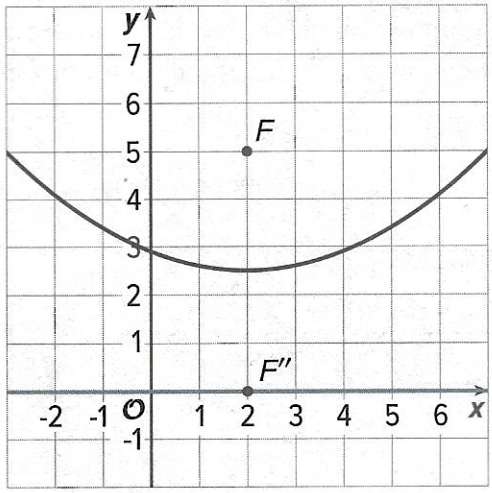
2. The point (14, 10) looks like it might be on the parabola. Determine if it really is on the parabola. Explain or show your reasoning.

3. In general, how can you determine if a particular point (x, y) is on the parabola?

NAME _____ DATE _____ PERIOD _____

Are you ready for more?

The image shows a parabola with directrix $y = 0$ and focus at $F = (2, 5)$.



Imagine you moved the focus from F to $F' = (2, 2)$.

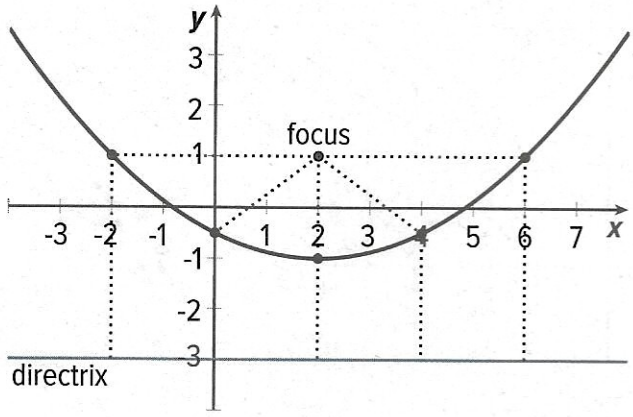
1. Sketch the new parabola.
2. How does decreasing the distance between the focus and the directrix change the shape of the parabola?

3. Suppose the focus were at F'', on the directrix. What would happen?

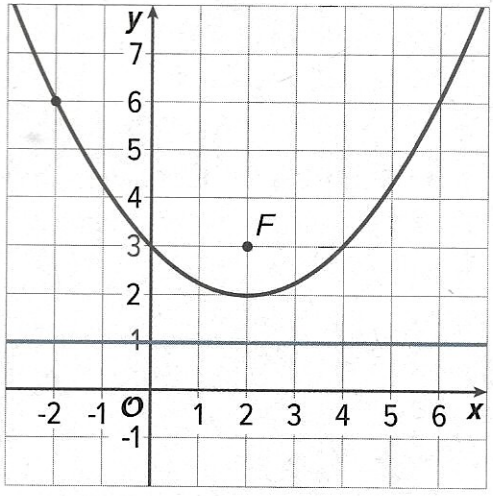
Summary

Distances and Parabolas

The diagram shows several points that are the same distance from the point (2, 1) as they are from the line $y = -3$. (Distance is measured from each point to the line along a segment perpendicular to the line.) The set of *all* points that are the same distance from a given point and a given line form a **parabola**. The given point is called the parabola's **focus** and the line is called its **directrix**.



We can use this definition to test if points are on a parabola. The image shows the parabola with focus (2, 3) and directrix $y = 1$. The point (-2, 6) appears to be on the parabola. Counting downwards, the distance between (-2, 6) and the directrix is 5 units.



Now use the Pythagorean Theorem to find the distance d between (-2, 6) and the focus, (2, 3). Imagine drawing a right triangle whose hypotenuse is the segment connecting (-2, 6) and (2, 3). The lengths of the triangle's legs can be found by subtracting the corresponding coordinates of the points.

Use those lengths in the Pythagorean Theorem to get $(-2 - 2)^2 + (6 - 3)^2 = d^2$. Evaluate the left side of the equation to find that $25 = d^2$. The distance, then, is 5 units because 5 is the positive number that squares to make 25. Now we know the point (-2, 6) really is on the parabola, because it's 5 units away from both the focus and the directrix.

Glossary

directrix
focus
parabola

NAME _____ DATE _____ PERIOD _____

Practice

Distances and Parabolas

1. The point $(6, y)$ is the same distance from (4, 1) as it is from the x-axis. What is the value of y?

2. A parabola is defined as the set of points the same distance from (6, 2) and the line $y = 4$. Select **all** points that are on this parabola.

   (A.) (1, -2) (C.) (6, 2) (E.) (8, 2)

   (B.) (2, -1) (D.) (7, 3)

3. Compare and contrast the parabolas with these definitions.
   - parabola A: points that are the same distance from (0, 4) and the x-axis
   - parabola B: points that are the same distance from (0, -6) and the x-axis

4. Find the center and radius of the circle represented by the equation $x^2 + y^2 - 8y + 5 = 0$. (Lesson 6-6)

5. Match each expression with the value needed in the box in order for the expression to be a perfect square trinomial. (Lesson 6-6)

A. $x^2 + 14x + \square$	1. $\frac{1}{16}$
B. $x^2 - \frac{1}{2}x + \square$	2. 49
C. $c^2 - 10c + \square$	3. 25
D. $z^2 + z + \square$	4. $\frac{1}{4}$

6. Write each expression as the square of a binomial. (Lesson 6-5)

   a. $x^2 - 12x + 36$

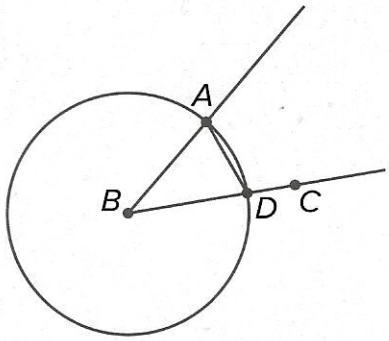
   b. $y^2 + 8y + 16$

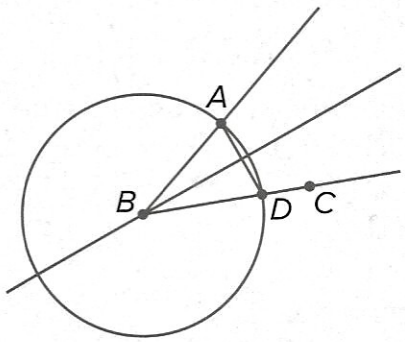
   c. $w^2 - 16w + 64$

7. Write an equation of a circle that is centered at (1, -4) with a radius of 10. (Lesson 6-4)

8. The density of water is 1 gram per cm^3. An object floats in water if its density is less than water's density, and it sinks if its density is greater than water's. Will a solid bar of soap shaped like a rectangular prism with mass 1.048 kilograms and dimensions 5.6 centimeters, 13 centimeters, and 16 centimeters float or sink? Explain your reasoning. (Lesson 5-17)

9. Jada has this idea for bisecting angle *ABC*. First she draws a circle with center *B* through *A*. Then she constructs the perpendicular bisector of *AD*.





Does Jada's construction work? Explain your reasoning. You may assume that the perpendicular bisector of line segment *AD* is the set of points equidistant from *A* and *D*. (Lesson 1-5)

Lesson 6-8

Equations and Graphs

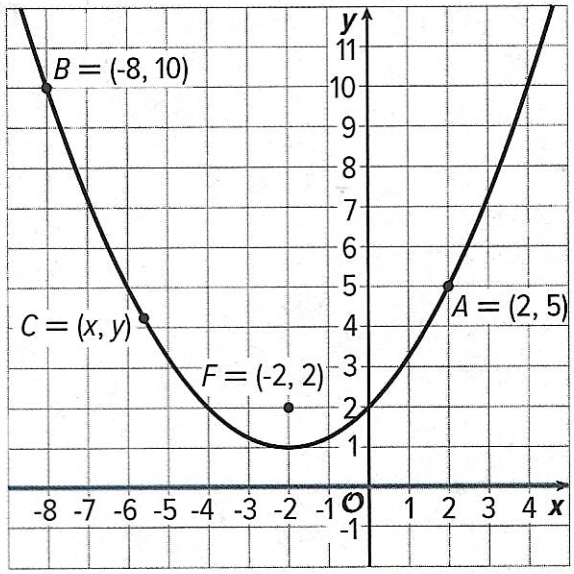
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's write an equation for a parabola.

Warm Up

8.1 Focus on Distance

The image shows a parabola with focus (-2, 2) and directrix $y = 0$ (the x-axis). Points A, B, and C are on the parabola.

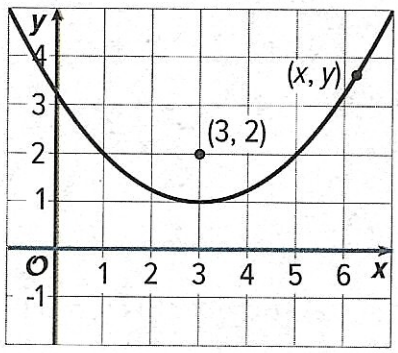


Without using the Pythagorean Theorem, find the distance from each plotted point to the parabola's focus. Explain your reasoning.

Activity

8.2 Building an Equation for a Parabola

The image shows a parabola with focus (3, 2) and directrix $y = 0$ (the x-axis).



1. Write an equation that would allow you to test whether a particular point (x, y) is on the parabola.

2. The equation you wrote defines the parabola, but it's not in a very easy-to-read form. Rewrite the equation to be in vertex form: $y = a(x - h)^2 + k$, where (h, k) is the vertex.

Activity

8.3 Card Sort: Parabolas

Your teacher will give you a set of cards with graphs and equations of parabolas. Match each graph with the equation that represents it.

NAME _____ DATE _____ PERIOD _____

Are you ready for more?

In this section, you have examined points that are equidistant from a given point and a given line. Now consider a set of points that are half as far from a point as they are from a line.

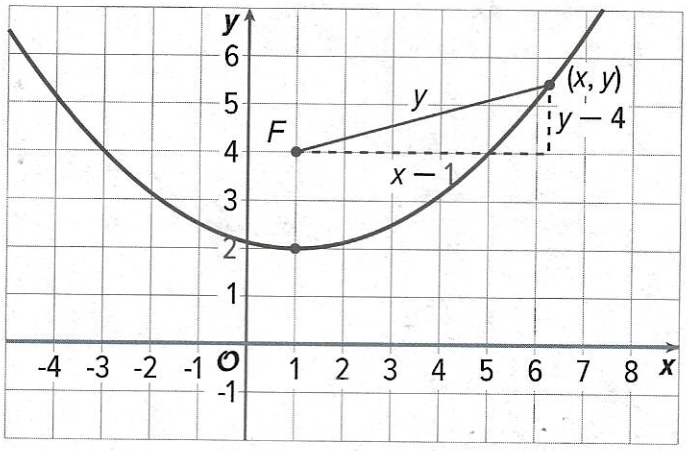
1. Write an equation that describes the set of all points that are $\frac{1}{2}$ as far from the point (5, 3) as they are from the x-axis.

2. Use technology to graph your equation. Sketch the graph and describe what it looks like.

Summary

Equations and Graphs

The parabola in the image consists of all the points that are the same distance from the point (1, 4) as they are from the line $y = 0$. Suppose we want to write an equation for the parabola—that is, an equation that says a given point (x, y) is on the curve. We can draw a right triangle whose hypotenuse is the distance between point (x, y) and the focus, (1, 4).



The distance from (x,y) to the directrix, or the line $y = 0$, is y units. By definition, the distance from (x,y) to the focus must be equal to the distance from the point to the directrix. So, the distance from (x,y) to the focus can be labeled with y. To find the lengths of the legs of the right triangle, subtract the corresponding coordinates of the point (x,y) and the focus, (1, 4). Substitute the expressions for the side lengths into the Pythagorean Theorem to get an equation defining the parabola.

$$(x-1)^2 + (y-4)^2 = y^2$$

To get the equation looking more familiar, rewrite it in vertex form, or $y = a(x-h)^2 + k$ where (h, k) is the vertex.

$$(x-1)^2 + (y-4)^2 = y^2$$

$$(x-1)^2 + y^2 - 8y + 16 = y^2$$

$$(x-1)^2 - 8y + 16 = 0$$

$$-8y = -(x-1)^2 - 16$$

$$y = \frac{1}{8}(x-1)^2 + 2$$

NAME _____ DATE _____ PERIOD _____

Practice

Equations and Graphs

1. Classify the graph of the equation $x^2 + y^2 - 8x + 4y = 29$.

   (A.) circle

   (B.) exponential curve

   (C.) line

   (D.) parabola

2. Write an equation that states (x, y) is the same distance from (4, 1) as it is from the x-axis.

3. Select **all** equations which describe the parabola with focus (-1, -7) and directrix $y = 3$.

   (A.) $(x - 1)^2 + (y - 7)^2 = (y + 3)^2$

   (B.) $(x + 1)^2 + (y + 7)^2 = (y - 3)^2$

   (C.) $y = -20(x + 1)^2 - 2$

   (D.) $y = -20(x + 1)^2 + 2$

   (E.) $y = -\frac{1}{20}(x + 1)^2 - 2$

   (F.) $y = -\frac{1}{20}(x + 1)^2 + 2$

4. Parabola A and parabola B both have the x-axis as the directrix. Parabola A has its focus at (3, 2) and parabola B has its focus at (5, 4). Select **all** true statements. (Lesson 6-7)

   (A.) Parabola A is wider than parabola B.

   (B.) Parabola B is wider than parabola A.

   (C.) The parabolas have the same line of symmetry.

   (D.) The line of symmetry of parabola A is to the right of that of parabola B.

   (E.) The line of symmetry of parabola B is to the right of that of parabola A.

5. A parabola has focus (5, 1) and directrix $y = -3$. Where is the parabola's vertex? (Lesson 6-7)

6. Select the value needed in the box in order for the expression to be a perfect square trinomial. (Lesson 6-6)

$x^2 + 7x + \square$

(A.) 3.5

(C.) 12.25

(B.) 7

(D.) 14.5

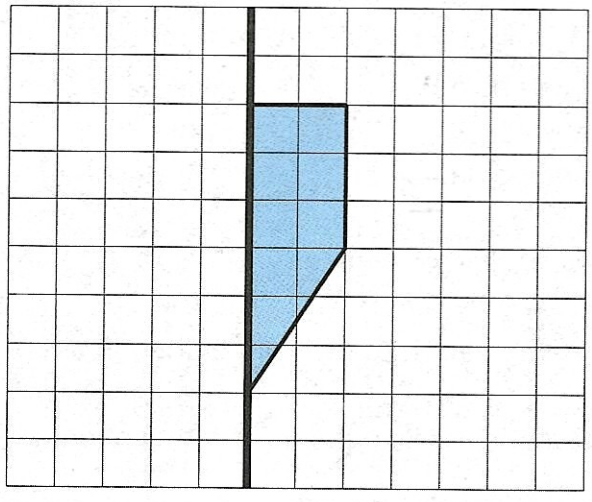
7. Rewrite each expression as the product of 2 factors. (Lesson 6-5)

a. $x^2 + 3x$

b. $x^2 - 6x - 7$

c. $x^2 + 4x + 4$

8. Suppose this two-dimensional figure is rotated 360 degrees using the vertical axis shown. Each small square on the grid represents 1 square inch. What is the volume of the three-dimensional figure? (Lesson 5-15)



Lesson 6-9

Equations of Lines

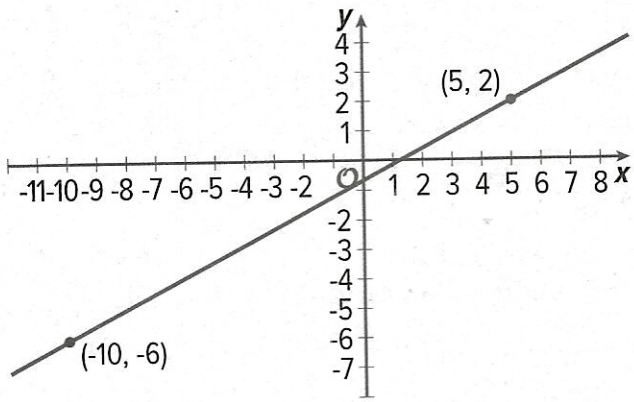
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's investigate equations of lines.

Warm Up

9.1 Remembering Slope

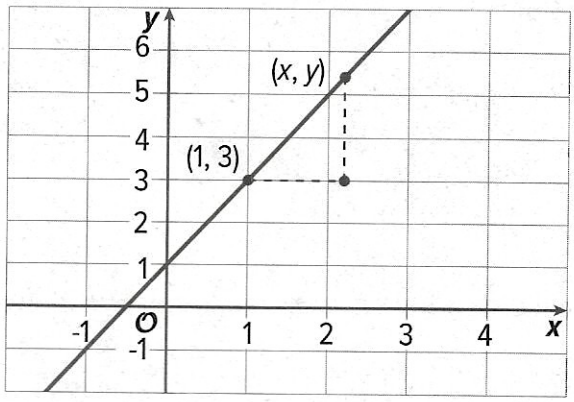
The slope of the line in the image is $\frac{8}{15}$. Explain how you know this is true.



Activity

9.2 Building an Equation for a Line

1. The image shows a line.



   a. Write an equation that says the slope between the points (1, 3) and (x, y) is 2.

   b. Look at this equation: $y - 3 = 2(x - 1)$
   How does it relate to the equation you wrote?

2. Here is an equation for another line: $y - 7 = \frac{1}{2}(x - 5)$

   a. What point do you know this line passes through?

   b. What is the slope of this line?

3. Next, let's write a general equation that we can use for any line. Suppose we know a line passes through a particular point (h, k).

   a. Write an equation that says the slope between point (x, y) and (h, k) is m.

   b. Look at this equation: $y - k = m(x - h)$. How does it relate to the equation you wrote?

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Activity

9.3 Using Point-Slope Form

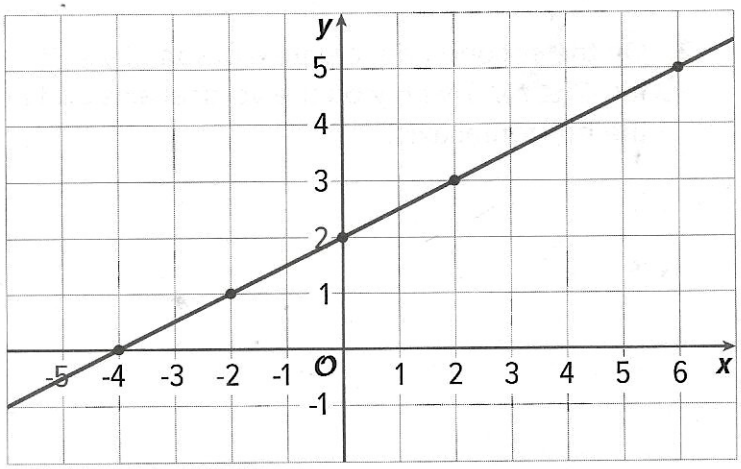
1. Write an equation that describes each line.

   a. the line passing through point $(-2, 8)$ with slope $\frac{4}{5}$

   b. the line passing through point $(0, 7)$ with slope $-\frac{7}{3}$

   c. the line passing through point $\left(\frac{1}{2}, 0\right)$ with slope -1

   d. the line in the image



2. Using the structure of the equation, what point do you know each line passes through? What's the line's slope?

   a. $y - 5 = \frac{3}{2}(x + 4)$

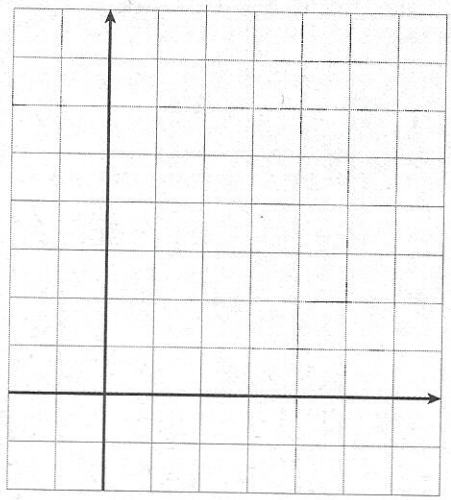
   b. $y + 2 = 5x$

   c. $y = -2\left(x - \frac{5}{8}\right)$

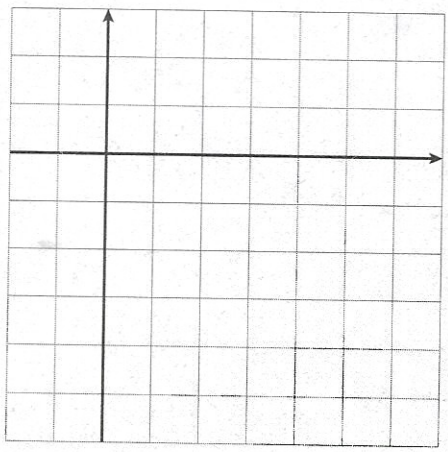
Are you ready for more?

Another way to describe a line, or other graphs, is to think about the coordinates as changing over time. This is especially helpful if we're thinking tracing an object's movement. This example describes the x- and y-coordinates separately, each in terms of time, t.

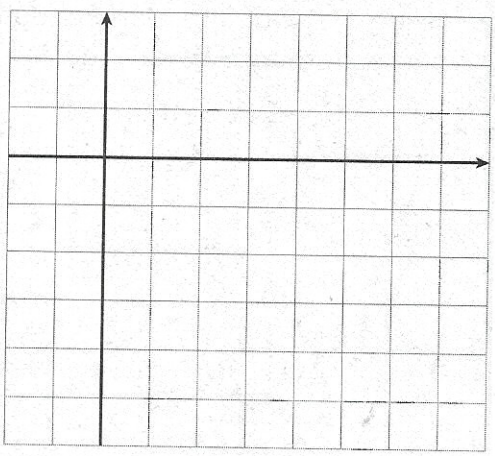
1. On the first grid, create a graph of $x = 2 + 5t$ for $-2 \leq t \leq 7$ with x on the vertical axis and t on the horizontal axis.



2. On the second grid, create a graph of $y = 3 - 4t$ for $-2 \leq t \leq 7$ with y on the vertical axis and t on the horizontal axis.



3. On the third grid, create a graph of the set of points $(2 + 5t, 3 - 4t)$ for $-2 \leq t \leq 7$ on the xy-plane.

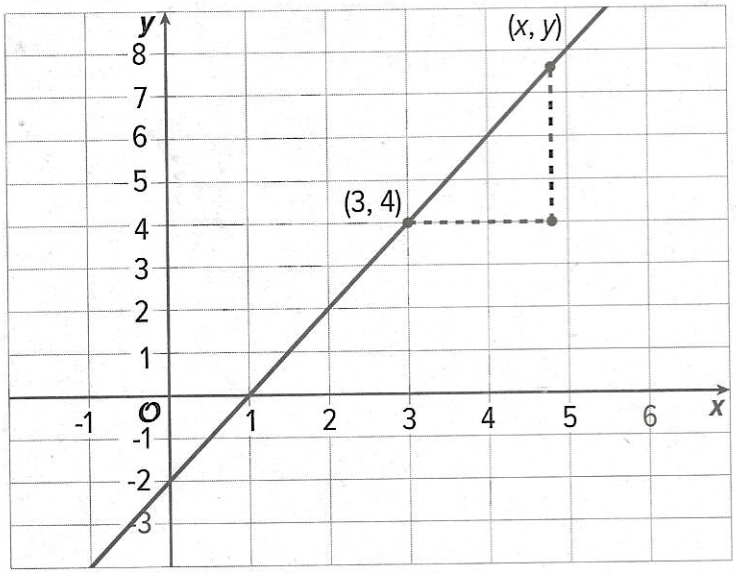


NAME _____ DATE _____ PERIOD _____

Summary

Equations of Lines

The line in the image can be defined as the set of points that have a slope of 2 with the point (3, 4). An equation that says point (x, y) has slope 2 with (3, 4) is $\frac{y-4}{x-3} = 2$. This equation can be rearranged to look like $y - 4 = 2(x - 3)$.



The equation is now in **point-slope form**, or $y - k = m(x - h)$, where:

- (x, y) is any point on the line
- (h, k) is a particular point on the line that we choose to substitute into the equation
- m is the slope of the line

Other ways to write the equation of a line include slope-intercept form, $y = mx + b$, and standard form, $Ax + By = C$.

To write the equation of a line passing through (3, 1) and (0, 5), start by finding the slope of the line. The slope is $-\frac{4}{3}$ because $\frac{5-1}{0-3} = -\frac{4}{3}$. Substitute this value for m to get $y - k = -\frac{4}{3}(x - h)$. Now we can choose any point on the line to substitute for (h, k). If we choose (3, 1), we can write the equation of the line as $y - 1 = -\frac{4}{3}(x - 3)$.

We could also use (0, 5) as the point, giving $y - 5 = -\frac{4}{3}(x - 0)$. We can rearrange the equation to see how point-slope and slope-intercept forms relate, getting $y = -\frac{4}{3}x + 5$. Notice (0, 5) is the y-intercept of the line.

The graphs of all 3 of these equations look the same.

Glossary

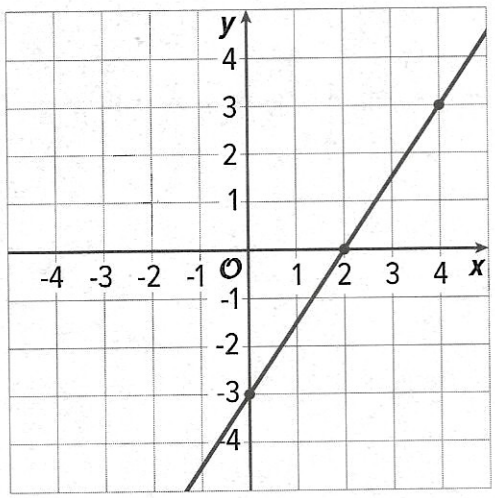
point-slope form

NAME _____ DATE _____ PERIOD _____

Practice

Equations of Lines

1. Select **all** the equations that represent the graph shown.



A. $3x - 2y = 6$

B. $y = \frac{3}{2}x + 3$

C. $y = \frac{3}{2}x - 3$

D. $y - 3 = \frac{3}{2}(x - 4)$

E. $y - 6 = \frac{3}{2}(x - 2)$

2. A line with slope $\frac{3}{2}$ passes through the point (1, 3).

   a. Explain why (3, 6) is on this line.

   b. Explain why (0, 0) is not on this line.

   c. Is the point (13, 22) on this line? Explain why or why not.

3. Write an equation of the line that passes through (1, 3) and has a slope of $\frac{5}{4}$.

4. A parabola has focus (3, -2) and directrix $y = 2$. The point $(a, -8)$ is on the parabola. How far is this point from the focus? (Lesson 6-8)

   (A.) 6 units

   (B.) 8 units

   (C.) 10 units

   (D.) cannot be determined

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5. Write an equation for a parabola with each given focus and directrix. **(Lesson 6-8)**

a. focus: (5, 2); directrix: x-axis

b. focus: (-2, 3); directrix: the line $y = 7$

c. focus: (0, 7); directrix: x-axis

d. focus: (-3, -4); directrix: the line $y = -1$

6. A parabola has focus (-1, 6) and directrix $y = 4$. Determine whether each point on the list is on this parabola. Explain your reasoning. **(Lesson 6-7)**

a. (-1, 5)

b. (1, 7)

c. (3, 9)

7. Select the center of the circle represented by the equation $x^2 + y^2 - 8x + 11y - 2 = 0$. (Lesson 6-6)

(A.) (8, 11)

(B.) (4, 5.5)

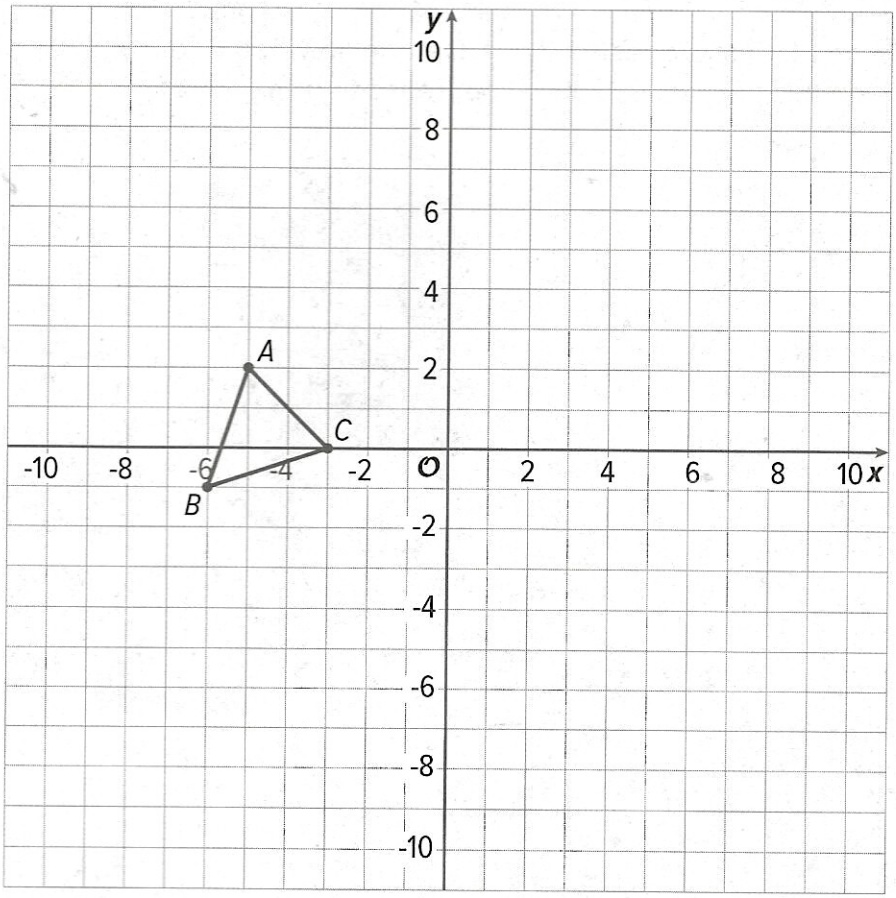
(C.) (-4, -5.5)

(D.) (4, -5.5)

8. Reflect triangle *ABC* over the line $x = -6$. (Lesson 6-1)

Translate the image by the directed line segment from (0, 0) to (5, -1).

What are the coordinates of the vertices in the final image?



Lesson 6-10

Parallel Lines in the Plane

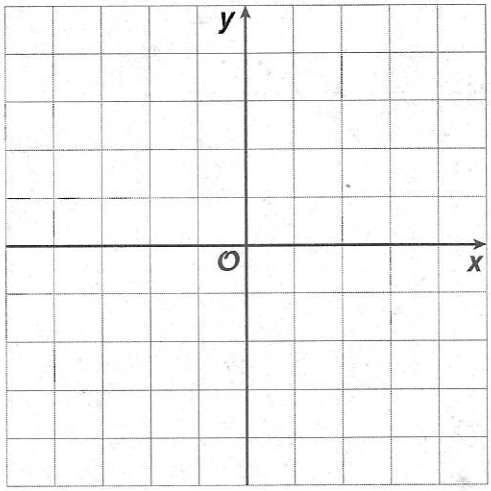
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's investigate parallel lines in the coordinate plane.

Warm Up

10.1 Translating Lines

1. Draw any non-vertical line in the plane. Draw 2 possible translations of the line.



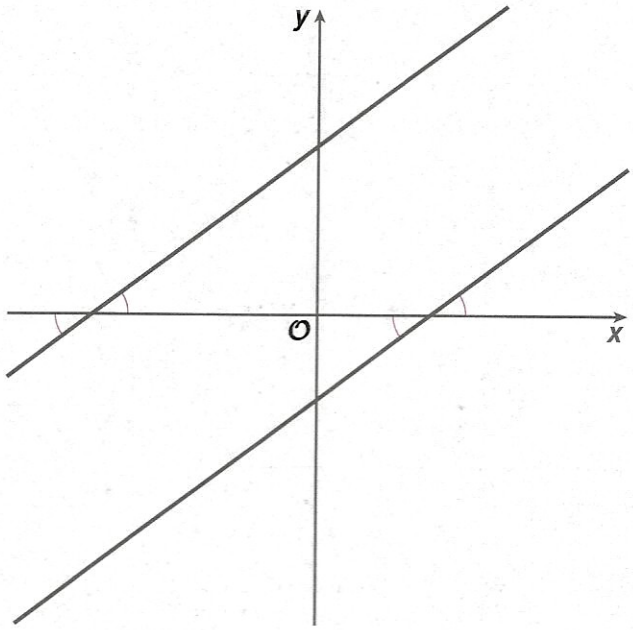
2. Find the slope of your original line and the slopes of the images.

Activity

10.2 Priya's Proof

Priya writes a proof saying:

Consider any 2 parallel lines. Assume they are not horizontal or vertical. Therefore they must pass through the x-axis as well as the y-axis. This forms 2 right triangles with a second congruent angle. Call the angle θ. The tangent of θ is equal for both triangles. Therefore the lines have the same slope.



1. How does Priya know the right triangles have a second congruent angle?

2. Show or explain what it means that the tangent of θ is equal for both triangles.

3. How does this prove the slopes of parallel lines are equal?

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Activity

10.3 Prove Your Parallelogram

1. Write the equation of a line parallel to $y = 2x + 3$, passing through (-4, 1).

2. Graph both lines described in the previous question.

3. Draw a parallelogram using the 2 lines you graphed and using (-4, 1) as one of the vertices.
4. Prove that your figure is a parallelogram.

Are you ready for more?

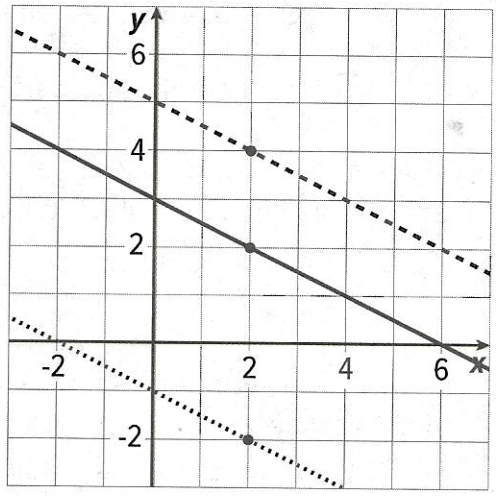
Prove algebraically that the translation $(x, y) \rightarrow (x + p, y + q)$ takes the line $y = mx + b$ to a line with the same slope.

Summary

Parallel Lines in the Plane

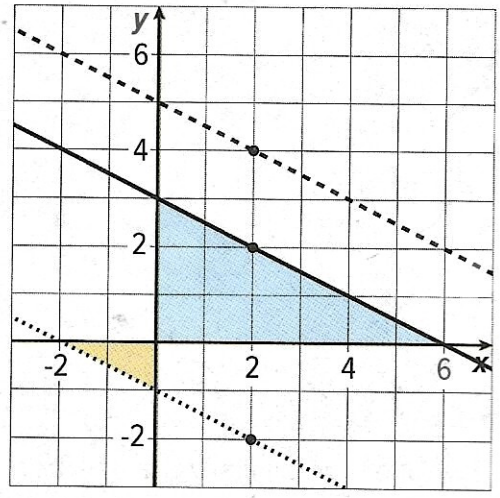
The solid line has been translated in 2 different ways:

1. by the directed line segment from (2, 2) to (2, 4) to produce the dashed line above
2. by the directed line segment from (2, 2) to (2, -2) to produce the dotted line below



The 3 lines look parallel to one another, as we would expect. We know that translations of lines result in parallel lines.

What happens to the slopes of these lines? If we draw in the slope triangles that go through the origin, we can see right triangles. Since we know the lines are parallel, the corresponding pairs of angles in the triangles must be congruent by the Alternate Interior Angles Theorem. Triangles with congruent angles are similar, and similar slope triangles result in lines with the same slope. Here we see slopes of $-\frac{5}{10}$, $-\frac{3}{6}$, and $-\frac{1}{2}$, which are all equal.



We can use similar reasoning to show that any 2 parallel lines that aren't vertical have the same slope, and also that any 2 lines with the same slope are parallel.

What if we wanted to find the equation of a line parallel to these 3 lines that goes through the point (6, -1)? We know the line must have the same slope of $-\frac{1}{2}$. We can use point-slope form and get $y + 1 = -\frac{1}{2}(x - 6)$.

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Practice

Parallel Lines in the Plane

1. Select **all** equations that are parallel to the line $2x + 5y = 8$.

   (A.) $y = \frac{2}{5}x + 4$

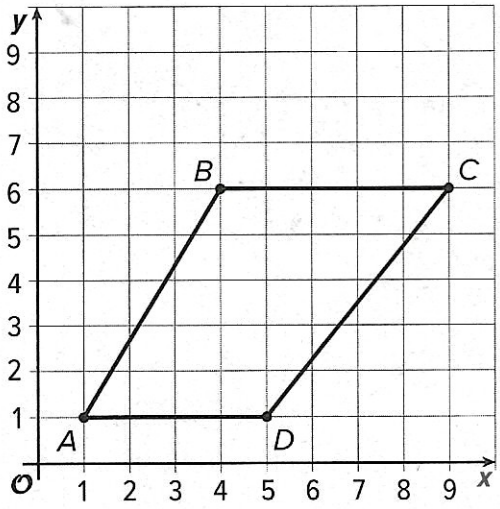
   (B.) $y = -\frac{2}{5}x + 4$

   (C.) $y - 2 = \frac{5}{2}(x + 1)$

   (D.) $y - 2 = -\frac{2}{5}(x + 1)$

   (E.) $10x + 5y = 40$

2. Prove that *ABCD* is not a parallelogram.



3. Write an equation of a line that passes through (-1, 2) and is parallel to a line with *x*-intercept (3, 0) and *y*-intercept (0, 1).

4. Write an equation of the line with slope $\frac{2}{3}$ that goes through the point (-2, 5). (Lesson 6-9)

5. Priya and Han each wrote an equation of a line with slope $\frac{1}{3}$ that passes through the point (1, 2). Priya's equation is $y - 2 = \frac{1}{3}(x - 1)$ and Han's equation is $3y - x = 5$. Do you agree with either of them? Explain or show your reasoning. (Lesson 6-9)

6. Match each equation with another equation whose graph is the same parabola. (Lesson 6-8)

A. $(x-3)^2 + (y-2)^2 = y^2$	1. $y = \frac{1}{8}(x-2)^2$
B. $(x-2)^2 + (y-3)^2 = (y+3)^2$	2. $y = \frac{1}{12}(x-2)^2$
C. $(x-3)^2 + (y-4)^2 = (y+2)^2$	3. $y = \frac{1}{4}(x-3)^2 + 1$
D. $(x-2)^2 + (y-2)^2 = (y+2)^2$	4. $y = \frac{1}{12}(x-3)^2 + 1$

7. A parabola is defined as the set of points the same distance from (-1, 3) and the line $y = 5$. Select the point that is on this parabola. (Lesson 6-7)

(A.) (-1, 3)

(B.) (0, 5)

(C.) (3, 0)

(D.) (0, 0)

8. Here are some transformation rules. For each rule, describe whether the transformation is a rigid motion, a dilation, or neither. (Lesson 6-2)

a. $(x, y) \rightarrow (2x, y + 2)$

b. $(x, y) \rightarrow (2x, 2y)$

c. $(x, y) \rightarrow (x + 2, y + 2)$

d. $(x, y) \rightarrow (x - 2, y)$

Lesson 6-11

Perpendicular Lines in the Plane

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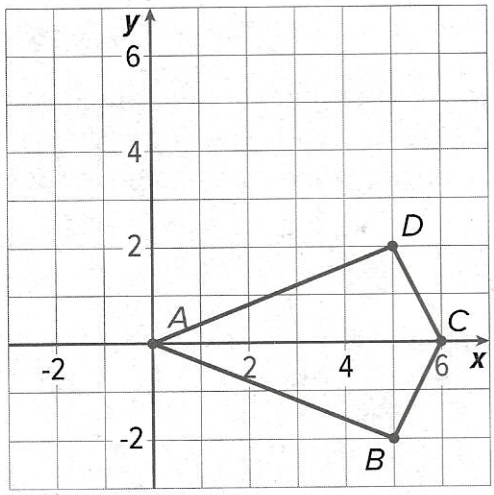
Learning Goal Let's analyze the slopes of perpendicular lines.

Warm Up

11.1 Revisiting Transformations

The image shows quadrilateral *ABCD*.



Apply the transformation rule $(x, y) \rightarrow (-y, x)$ to quadrilateral *ABCD*. What is the effect of the transformation rule?

Activity

11.2 Make a Conjecture

1. Complete the table with the slope of each segment from the warm-up.

	Original Figure Slope	Image Slope	Product
AB			
BC			
CD			
DA			

2. The image in the warm-up is a 90-degree rotation of the original figure, so each line in the original figure is perpendicular to the corresponding line in the image. Use your slope calculations to make a conjecture about slopes of perpendicular lines.

Activity

11.3 Prove It

Let's prove our conjecture about slopes of perpendicular lines for the case where the lines pass through the origin.

1. Find the slope of a line passing through the point (a, b) and the origin. Assume the line is not horizontal or vertical.

2. Suppose the line is rotated using the transformation rule $(x, y) \rightarrow (-y, x)$ Find the coordinates of the images of the points (a, b) and the origin.

3. How does the original line relate to the image?

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4. Find the slope of the image.

5. Compare your slopes. What did you just prove?

Are you ready for more?

We can take any rhombus *EFGH* and translate it so that the image of *E* is (0, 0). We can then rotate using (0, 0) as a center so that the image of *F* under this sequence is on the positive *x*-axis at some point (*c*, 0).

1. Suppose these transformations take the point *H* to some point (*a*, *b*). What must the coordinates of *G* be?
2. For the values of *a*, *b*, and *c* in this problem, why is it true that $a^2 + b^2 = c^2$?

3. Prove that the diagonals of the image are perpendicular.

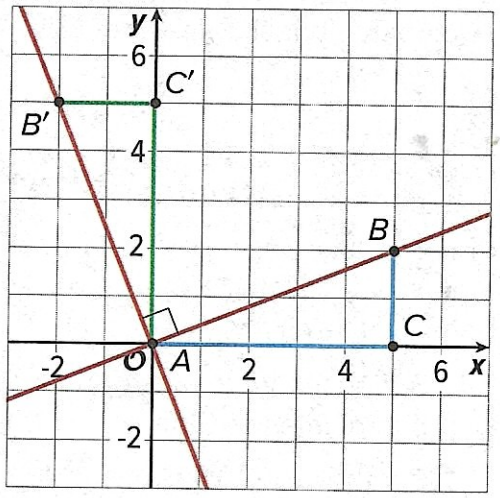
4. Prove that the diagonals of the image bisect each other.

5. Why does this imply the diagonals of **all** rhombi are perpendicular bisectors of each other?

Summary

Perpendicular Lines in the Plane

The diagram shows triangle ABC and its image, triangle $AB'C'$, under a 90-degree rotation counterclockwise using the origin as the center. Since the rotation was through 90 degrees, all line segments in the image are perpendicular to the corresponding segments in the original triangle. For example, segment AC is horizontal, while segment AC' is vertical.



Look at segments AB and AB', which, like the other pairs of segments, are perpendicular. The slope of segment AB is $\frac{2}{5}$, while the slope of segment AB' is $-\frac{5}{2}$. Notice the relationship between the slopes: They are reciprocals of one another, and have opposite signs. The product of the slopes, $\frac{2}{5} \cdot \left(-\frac{5}{2}\right)$, is -1. As long as perpendicular lines are not horizontal or vertical, their slopes will be **opposite reciprocals** and have a product of -1.

We can use this fact to help write equations of lines. For example, try writing the equation of a line that passes through the point (23, -30) and is perpendicular to a line ℓ represented by $y = 3x + 5$. The slope of line ℓ is 3. The slope of any line perpendicular to line ℓ is the opposite reciprocal of 3, or $-\frac{1}{3}$. Substitute the point (23, -30) and the slope $-\frac{1}{3}$ into the point-slope form to get the equation $y - (-30) = -\frac{1}{3}(x - 23)$.

Glossary

opposite reciprocal

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Practice

Perpendicular Lines in the Plane

1. Write an equation for a line that passes through the origin and is perpendicular to $y = 5x - 2$.

2. Match each line with a perpendicular line.

   A. $y = 5x + 2$

   B. $y - 2.25 = -2(x - 2)$

   C. the line through (-1, 5) and (1, 9)

   1. the line through (2, 12) and (17, 9)
   2. $y = -\frac{1}{2}x + 5$
   3. $2x - 4y = 10$

3. The rule $(x, y) \rightarrow (y, -x)$ takes a line to a perpendicular line. Select **all** the rules that take a line to a perpendicular line.

   A. $(x, y) \rightarrow (2y, -x)$

   B. $(x, y) \rightarrow (-y, -x)$

   C. $(x, y) \rightarrow (-y, x)$

   D. $(x, y) \rightarrow (0.5y, -2x)$

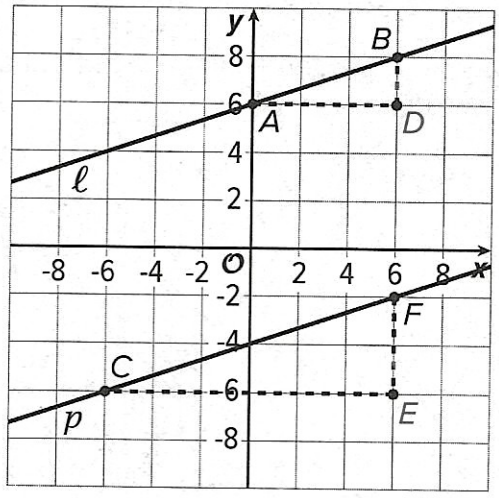
   E. $(x, y) \rightarrow (4y, -4x)$

4. Respond to each question. (Lesson 6-10)

   a. Write an equation of the line with x-intercept (3, 0) and y-intercept (0, -4).

   b. Write an equation of a line parallel to the line $y - 5 = \frac{4}{3}(x - 2)$.

5. Lines ℓ and p are parallel. Select **all** true statements. (Lesson 6-10)



(A.) Triangle ADB is similar to triangle CEF.

(B.) Triangle ADB is congruent to triangle CEF.

(C.) The slope of line ℓ is equal to the slope of line p.

(D.) $\sin(A) = \sin(C)$

(E.) $\sin(B) = \cos(C)$

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6. Select the equation that states (x, y) is the same distance from $(0, 5)$ as it is from the line $y = -3$. **(Lesson 6-8)**

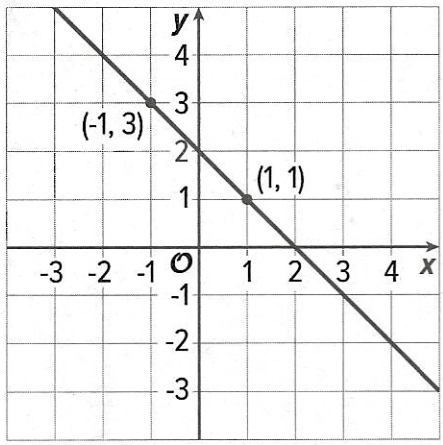
(A.) $x^2 + (y + 5)^2 = (y + 3)^2$

(B.) $x^2 + (y - 5)^2 = (y + 3)^2$

(C.) $x^2 + (y + 5)^2 = (y - 3)^2$

(D.) $x^2 + (y - 5)^2 = (y - 3)^2$

7. Select **all** equations that represent the graph shown. **(Lesson 6-9)**



(A.) $y = -x + 2$

(B.) $(y - 3) = -(x + 1)$

(C.) $(y - 3) = -x - 1$

(D.) $(y - 3) = (x - 1)$

(E.) $(y + 1) = -(x - 3)$

8. Write a rule that describes this transformation. (Lesson 6-3)

Original Figure	Image
(3, 2)	(6, 4)
(4, -1)	(8, -2)
(5, 1)	(10, 2)
(7, 3)	(14, 6)

Lesson 6-12

It's All on the Line

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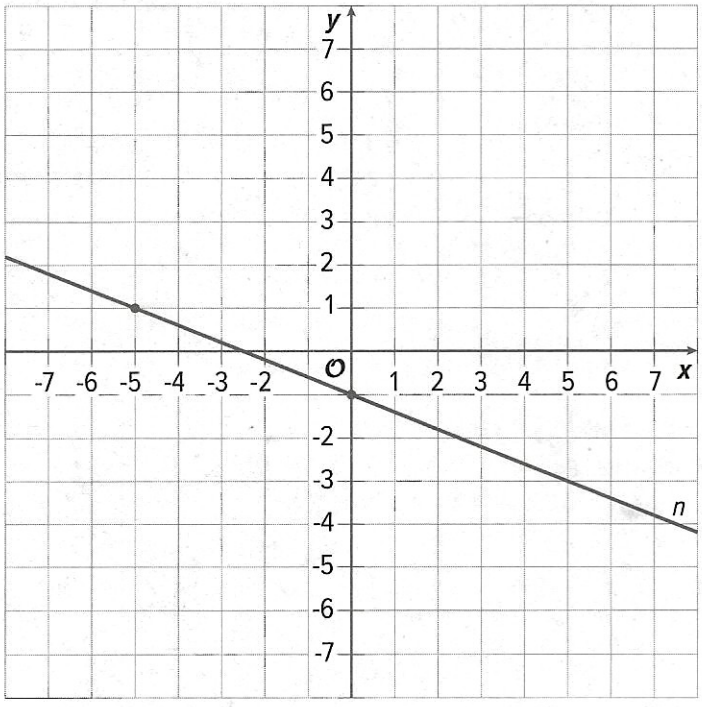
Learning Goal Let's work with both parallel and perpendicular lines.

Warm Up

12.1 Parallel and Perpendicular

The image shows line *n*.



1. Write an equation for the line that is perpendicular to *n* and whose *y*-intercept is (0, 5). Graph this line.

2. Write an equation for the line that is parallel to *n* and that passes through the point (3, 1). Graph this line.

Activity

12.2 Info Gap: Lines

Your teacher will give you either a problem card or a data card. Do not show or read your card to your partner.

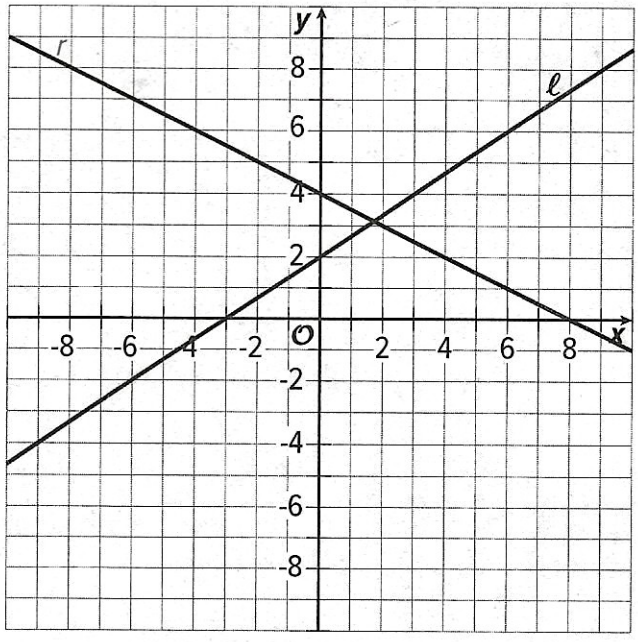
If your teacher gives you the data card:

1. Silently read the information on your card.
2. Ask your partner, "What specific information do you need?" and wait for your partner to ask for information. Only give information that is on your card. (Do not figure out anything for your partner!)
3. Before telling your partner the information, ask, "Why do you need to know (that piece of information)?"
4. Read the problem card, and solve the problem independently.
5. Share the data card, and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

If your teacher gives you the problem card:

1. Silently read your card and think about what information you need to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain to your partner how you are using the information to solve the problem.
4. When you have enough information, share the problem card with your partner, and solve the problem independently.
5. Read the data card, and discuss your reasoning.



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Activity

12.3 Three Lines

1. Line ℓ is represented by the equation $y = \frac{2}{3}x + 3$. Write an equation of the line perpendicular to ℓ, passing through (-6, 4). Call this line p.

2. Write an equation of the line perpendicular to p, passing through (3, -2). Call this line n.

3. What do you notice about lines ℓ and n? Does this always happen? Show or explain your answer.

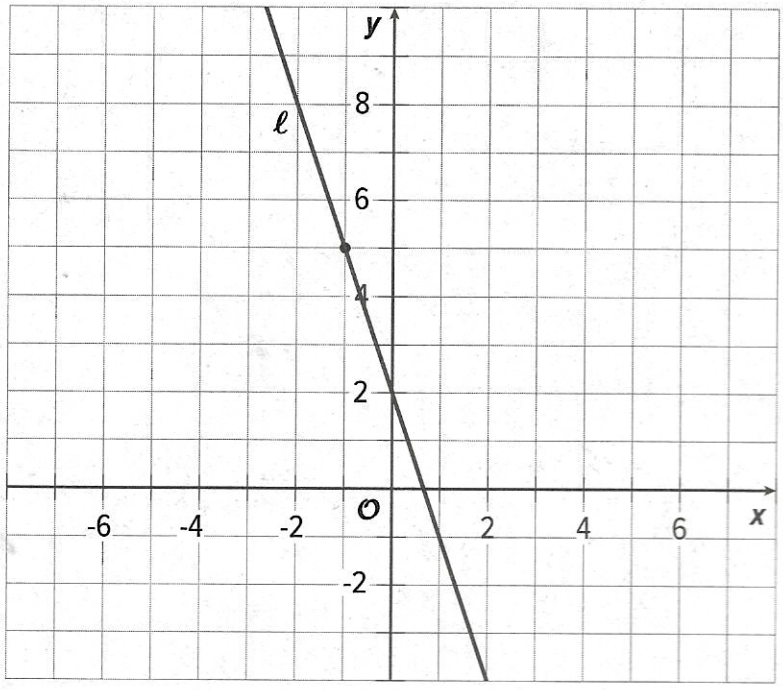
Are you ready for more?

Prove that the line $Ax + By = C$ is always perpendicular to the line that passes through (A, B) and the origin.

Summary

It's All on the Line

We can use the concepts of parallel and perpendicular lines to write equations of lines. The image shows line ℓ.



Suppose n is the image of ℓ when it is rotated 90 degrees using (-1, 5) as a center. What is an equation of line n?

The point (-1, 5) is on line ℓ. The center of rotation does not move when a figure is rotated, so (-1, 5) will also be on the image, line n. Because line ℓ was rotated 90 degrees, lines ℓ and n are perpendicular. Their slopes must be opposite reciprocals. The slope of line ℓ is -3, so the slope of n is $\frac{1}{3}$. Now substitute the slope $\frac{1}{3}$ and the point (-1, 5) into point-slope form to get $y - 5 = \frac{1}{3}(x - (-1))$.

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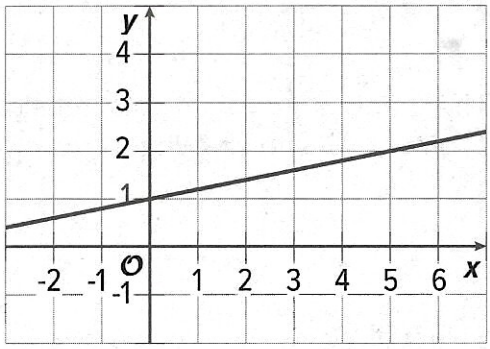
Practice

It's All on the Line

1. For each equation, is the graph of the equation parallel to the line shown, perpendicular to the line shown, or neither?



   a. $y = 0.2x$

   b. $y = -2x + 1$

   c. $y = 5x - 3$

   d. $(y - 3) = -5(x - 4)$

   e. $(y - 1) = 2(x - 3)$

   f. $5x + y = 3$

2. Main Street is parallel to Park Street. Park Street is parallel to Elm Street. Elm is perpendicular to Willow. How does Willow compare to Main?

3. The line which is the graph of $y = 2x - 4$ is transformed by the rule $(x, y) \rightarrow (-x, -y)$. What is the slope of the image?

4. Select **all** equations whose graphs are lines perpendicular to the graph of $3x + 2y = 6$ (Lesson 6-11)

   (A.) $3x - 2y = 4$

   (B.) $2x + 3y = 6$

   (C.) $2x - 3y = 8$

   (D.) $(y - 4) = \frac{2}{3}(x - 6)$

   (E.) $(y - 2) = -\frac{3}{2}(x - 8)$

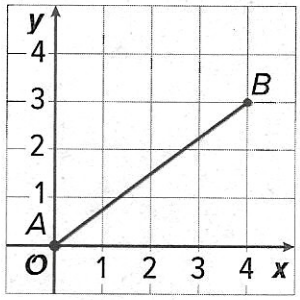
   (F.) $y = \frac{2}{3}x$

   (G.) $y = \frac{3}{2}x + 3$

5. Match each line with a perpendicular line. (Lesson 6-11)

A. the line through (12, 4) and (9, 19)	1. the line through (3, 1) and (1, 4)
B. $2x - 5y = 10$	2. $y = \frac{1}{5}x + 7$
C. $y - 4 = \frac{2}{3}(x + 1)$	3. $y - 1 = -2.5(x + 3)$

6. The graph of $y = -4x + 2$ is translated by the directed line segment AB shown. What is the slope of the image? (Lesson 6-10)



7. Select **all** points on the line with a slope of $-\frac{1}{2}$ that go through the point (4, -1) (Lesson 6-9)

A. (-2, 2)

B. (0, 2)

C. (4, -1)

D. (0, 1)

E. (-3, 8)

8. One way to define a circle is that it is the set of all points that are the same distance from a given center. How does the equation $(x - h)^2 + (y - k)^2 = r^2$ relate to this definition? Draw a diagram if it helps you explain. (Lesson 6-4)

Lesson 6-13

DATE _____ PERIOD _____

Intersection Points

NAME _____

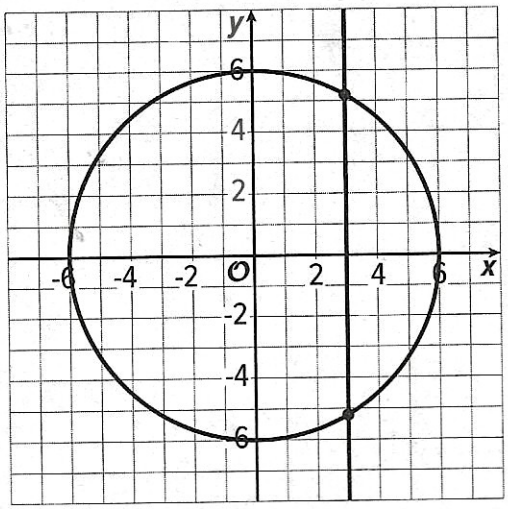
Learning Goal Let's look at how circles and parabolas interact with lines.

Warm Up

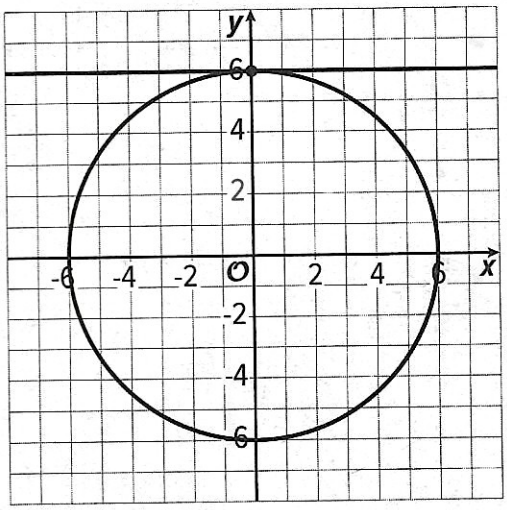
13.1 Which One Doesn't Belong: Lines and Curves

Which one doesn't belong?

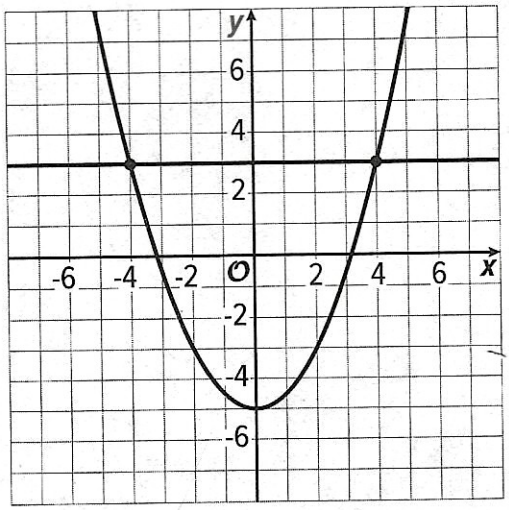
A



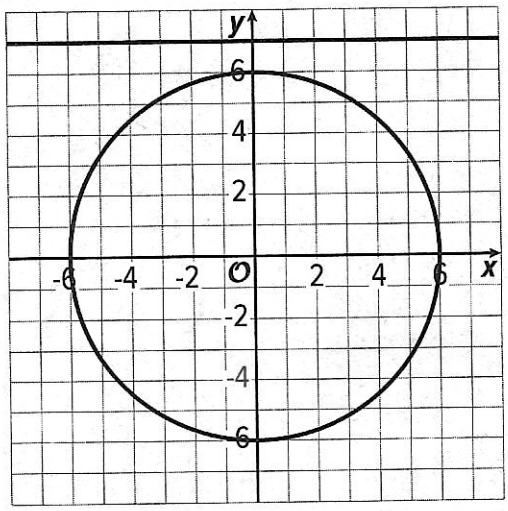
B



C

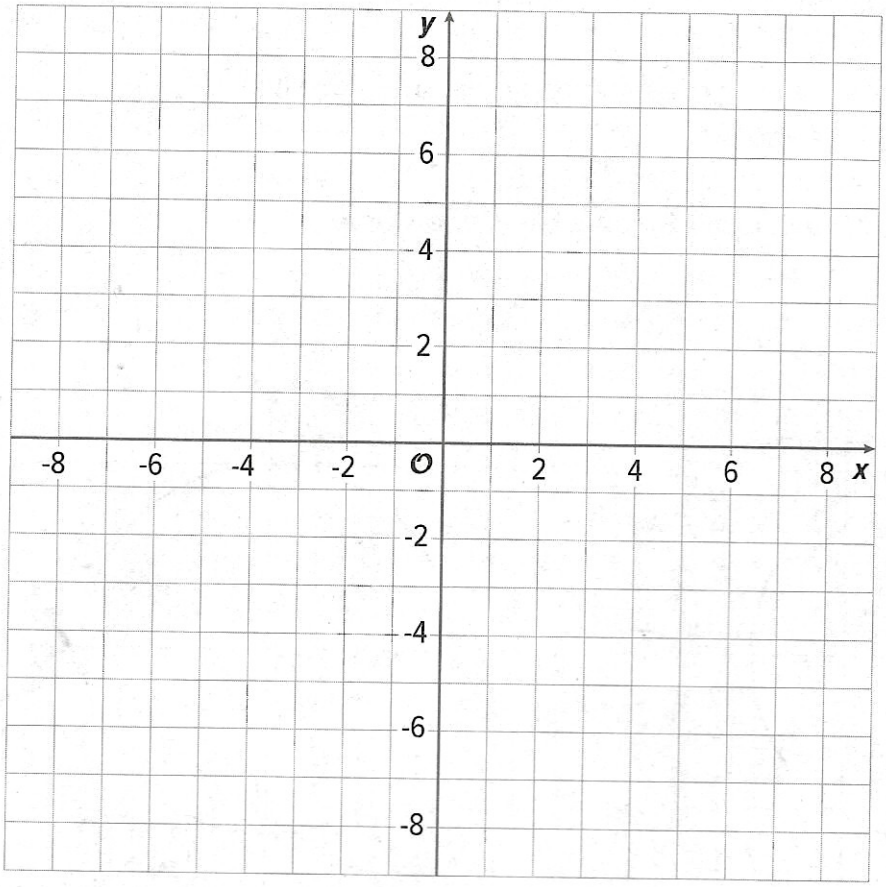


D



Activity

13.2 Circles and Lines



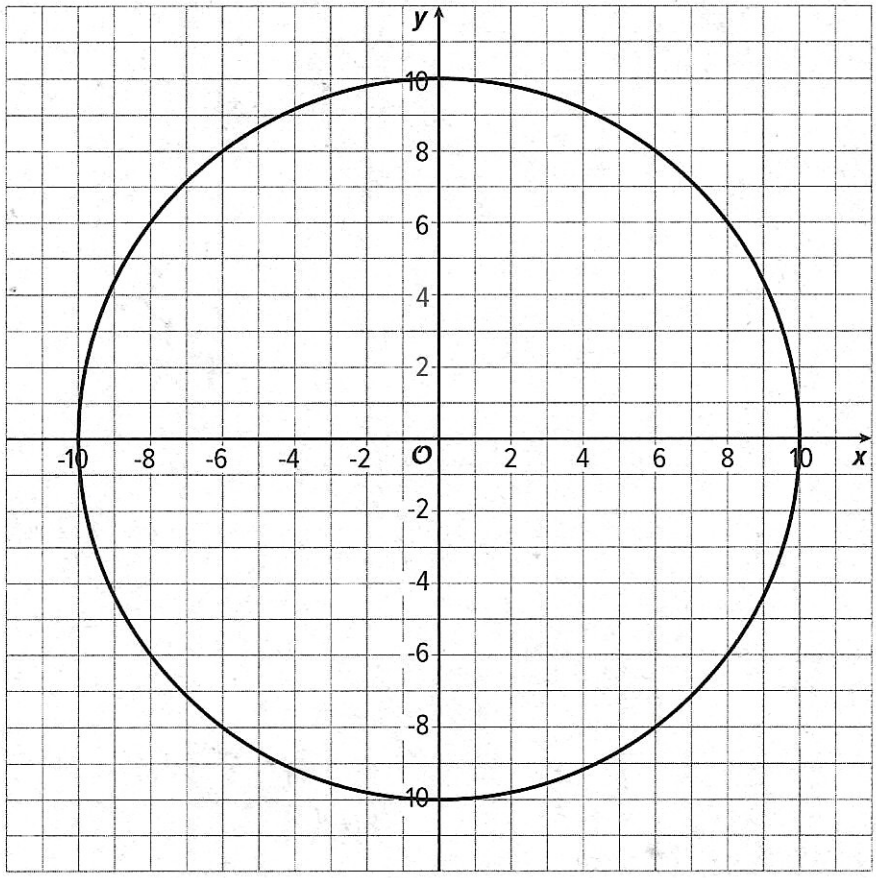
1. The equation $(x - 3)^2 + (y - 2)^2 = 25$ represents a circle. Graph this circle on the coordinate grid.
2. Graph the line $y = 6$. At what points does this line appear to intersect the circle?
3. How can you verify that the 2 figures really intersect at these points? Carry out whatever procedure you decide.

4. Graph the line $y = x - 2$. At what points does this line appear to intersect the circle? Verify that the 2 figures really do intersect at these points.

NAME _____ DATE _____ PERIOD _____

Activity

13.3 Creating Lines



1. Write an equation representing the circle in the graph.

2. Graph and write equations for each line described:

   a. any line parallel to the x-axis that intersects the circle at 2 points

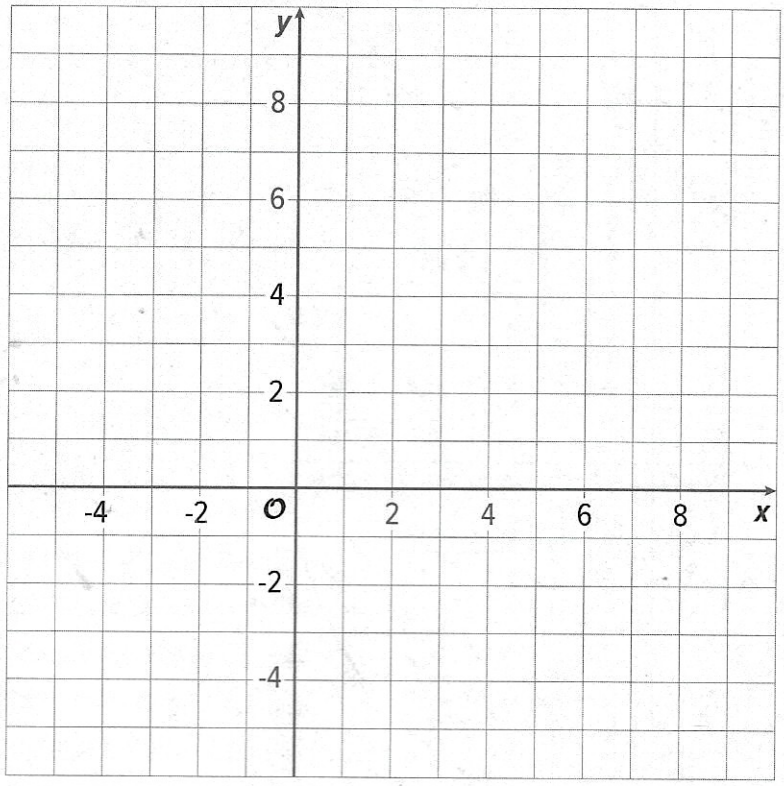
   b. any line perpendicular to the x-axis that doesn't intersect the circle

   c. the line perpendicular to $y = -\frac{1}{3}x + 5$ that intersects the circle at (6,8)

3. For the last line you graphed, find the second point where the line intersects the circle. Explain or show your reasoning.

Are you ready for more?

1. Graph the equations $y - 3 = (x - 2)^2$ and $y - 4 = 2(x - 3)$ and find their point of intersection.



2. Show that the graph of $y - 3 = (x - 2)^2$ and $y - 4 = m(x - 3)$ intersect at the point $(m + 1, m^2 - 2m + 4)$.

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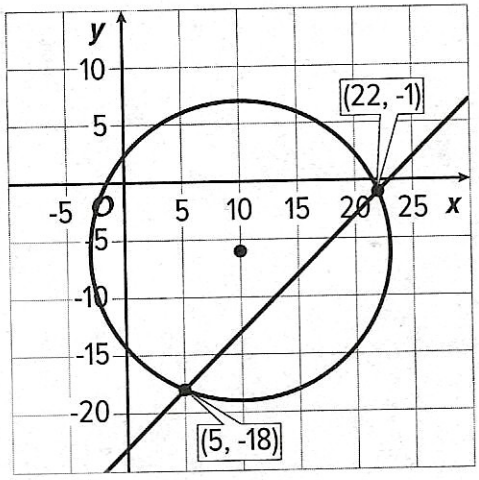
Summary

Intersection Points

We can graph circles and lines on the same coordinate grid and estimate where they intersect. The image shows the circle $(x - 10)^2 + (y + 6)^2 = 169$ and the line $y = x - 23$. The 2 figures appear to intersect at the points (22, -1) and (5, −18). To verify whether these truly are intersection points, we can check if substituting them into each equation produces true statements.

Let's test (22, -1). First, substitute it into the equation for the line. When we do so, we get $-1 = 22 - 23$. This is a true statement, so this point is on the line.

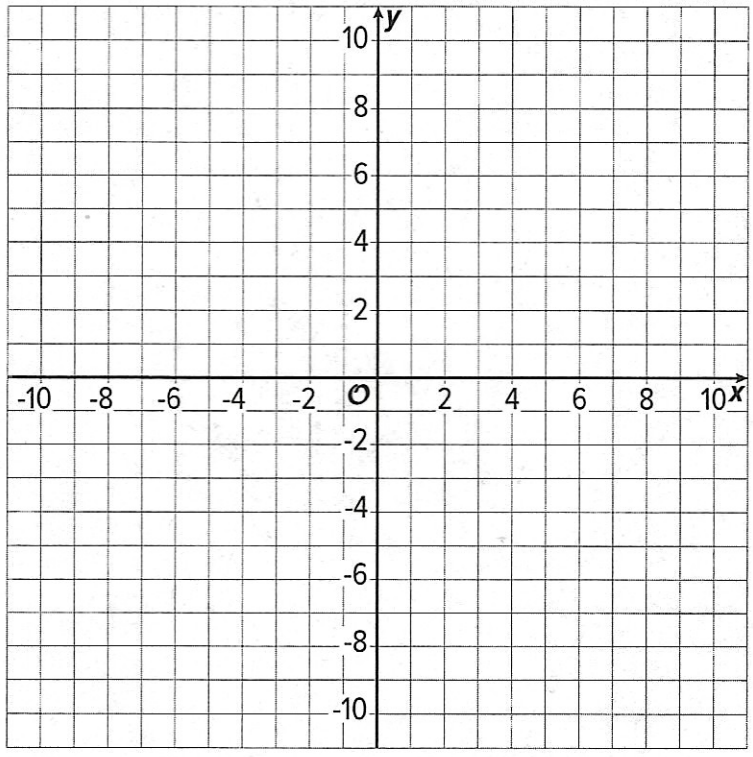


Next, substitute it into the equation for the circle. This is the same as checking to see if the distance from the point to the center is $\sqrt{169}$, or 13 units. We get $(22 - 10)^2 + (-1 - (-6))^2 = 169$. Evaluate the left side to get $144 + 25 = 169$. This is a true statement, so the point (22, -1) is on the circle. It's on both the circle and the line, so it must be an intersection point for the 2 figures.

Practice

Intersection Points

1. Graph the equations $(x - 2)^2 + (y + 3)^2 = 36$ and $x = 2$. Where do they intersect?



2. Select **all** equations for which the point (2, -3) is on the graph of the equation.

   (A.) $y - 3 = x - 2$

   (B.) $4x + y = 5$

   (C.) $y = 5x - 13$

   (D.) $x^2 + y^2 = 13$

   (E.) $(x - 2)^2 + (y - (-3))^2 = 25$

   (F.) $y = (x - 2)^2 + 3$

   (G.) $y = x^2 - 7$

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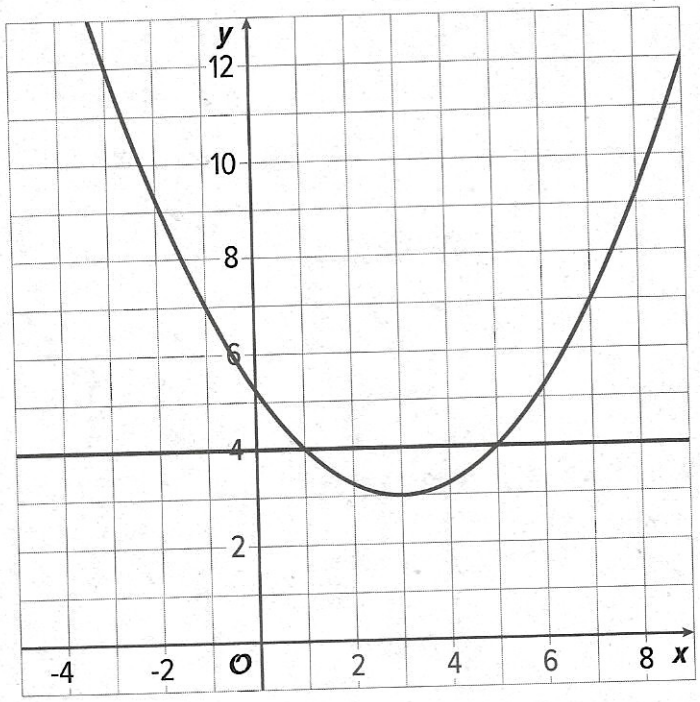
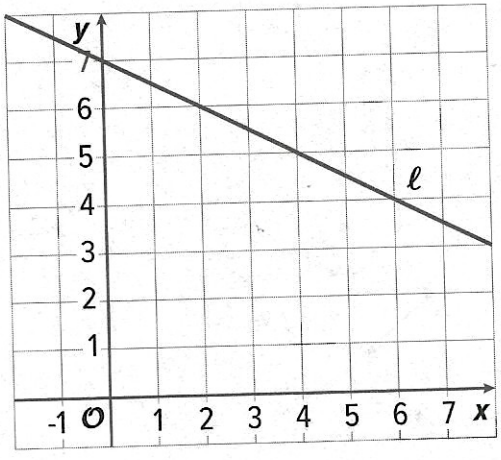
3. The image shows a graph of the parabola with focus (3, 4) and directrix $y = 2$, and the line given by $y = 4$. Find and verify the points where the parabola and the line intersect.



4. Here is a line ℓ. Write equations for and graph 4 different lines perpendicular to ℓ. (Lesson 6-12)



5. Write an equation whose graph is a line perpendicular to the graph of $y = 4$ and which passes through the point (2, 5). (Lesson 6-12)

6. Select **all** lines that are perpendicular to $y - 4 = -\frac{2}{3}(x + 1)$. (Lesson 6-11)

   (A.) $y = \frac{3}{2}x + 8$

   (B.) $3x - 2y = 2$

   (C.) $3x + 2y = 10$

   (D.) $y - 2 = -\frac{2}{3}(x - 1)$

   (E.) $y = \frac{3}{2}x$

7. Select the line parallel to $3x - 2y = 10$. (Lesson 6-10)

   (A.) $y - 1 = \frac{3}{2}(x + 6)$

   (B.) $6x + 4y = -20$

   (C.) $y = -\frac{3}{2}x + 2$

   (D.) $y - 4 = \frac{2}{3}(x + 1)$

8. Explain how you could tell whether $x^2 + bx + c$ is a perfect square trinomial. (Lesson 6-5)

Lesson 6-14

Coordinate Proof

NAME _____ DATE _____ PERIOD _____

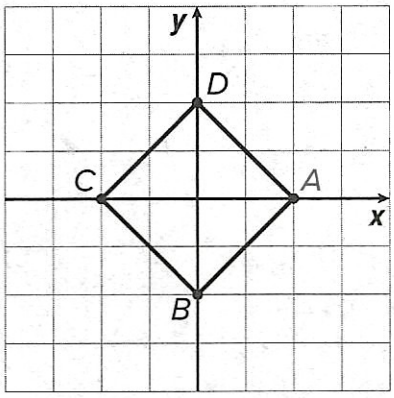
Learning Goals Let's use coordinates to prove theorems and to compute perimeter and area.

Warm Up

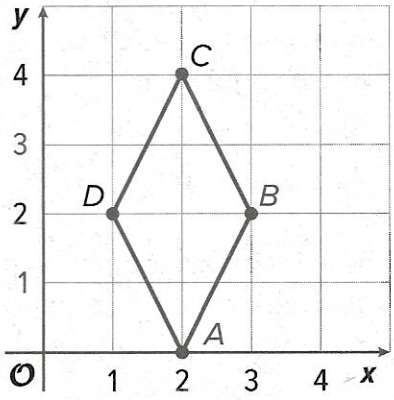
14.1 Which One Doesn't Belong: Coordinate Quadrilaterals

Which one doesn't belong?

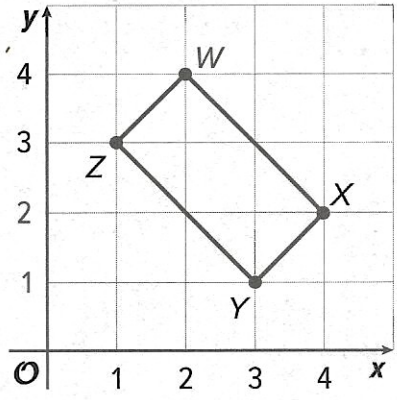
A



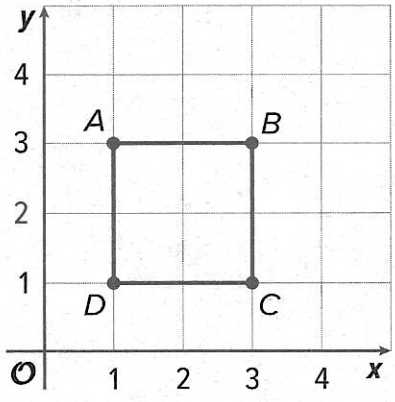
B



C



D



Activity

14.2 Name This Quadrilateral

A quadrilateral has vertices (0, 0), (4, 3), (13, -9), and (9, -12)

1. What type of quadrilateral is it? Explain or show your reasoning.

2. Find the perimeter of this quadrilateral.
3. Find the area of this quadrilateral.

Are you ready for more?

1. A parallelogram has vertices (0, 0), (5, 1), (2, 3), and (7, 4). Find the area of this parallelogram.

2. Consider a general parallelogram with vertices (0, 0), (a, b), (c, d), and $(a + c, b + d)$, where a, b, c, and d are positive. Write an expression for its area in terms of a, b, c, and d.

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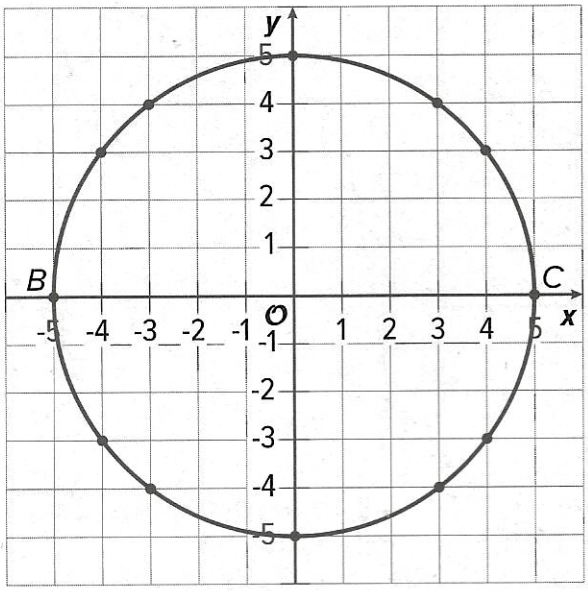
Activity

14.3 Circular Logic

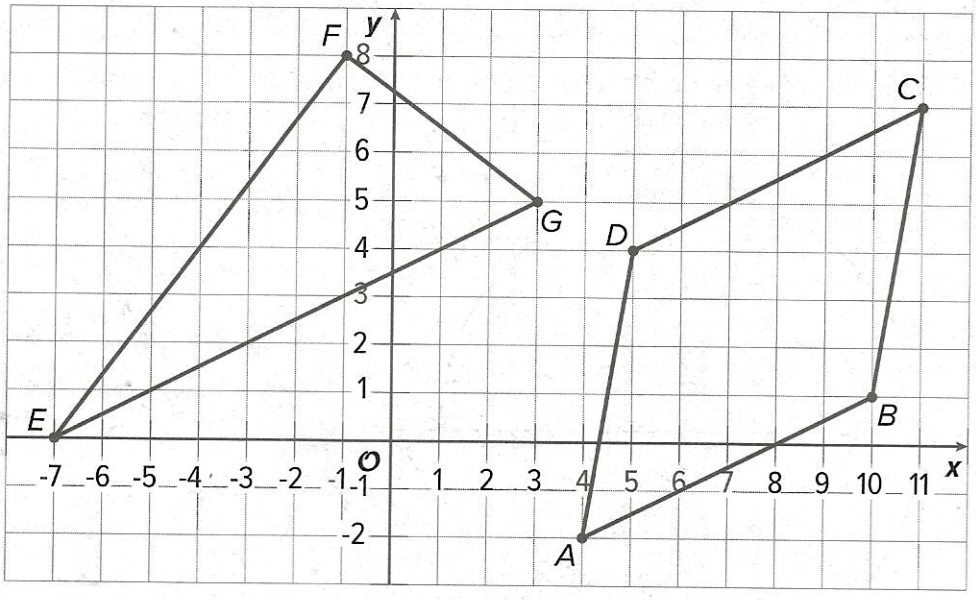
The image shows a circle with several points plotted on the circle.



1. What kind of segment is *BC* in reference to the circle?
2. Choose one of the plotted points on the circle and call it *D*. Each student in the group should choose a different point. Draw segments *BD* and *DC*. What does the measure of angle *BDC* appear to be?
3. Calculate the slopes of segments *BD* and *DC*. What do your results tell you?
4. Compare your results to those of others in your group. What did they find?
5. Based on your group's results, write a conjecture about these kinds of angles in circles.

Summary

Coordinate Proof



What kind of shape is quadrilateral *ABCD*? It looks like it might be a rhombus. To check, we can calculate the length of each side. Using the Pythagorean Theorem, we find that the lengths of segments *AB* and *CD* are $\sqrt{45}$ units, and the lengths of segments *BC* and *DA* are $\sqrt{37}$ units. All side lengths are between 6 and 7 units long, but they are not exactly the same. So our calculations show that *ABCD* is not really a rhombus, even though at first glance we might think it is.

We did just show that two pairs of opposite sides of *ABCD* are congruent. This means that *ABCD* must be a parallelogram. Checking slopes confirms this.

Sides *AB* and *CD* each have slope $\frac{1}{2}$. Sides *BC* and *DA* each have slope 6.

Can we find the area of triangle *EFG*? That seems tricky, because we don't know the height of the triangle using *EG* as the base. However, angle *EFG* seems like it could be a right angle. In that case, we could use sides *EF* and *FG* as the base and height.

To see if *EFG* is a right angle, we can calculate slopes. The slope of *EF* is $\frac{8}{6}$ or $\frac{4}{3}$, and the slope of *FG* is $-\frac{3}{4}$. Since the slopes are opposite reciprocals, the segments are perpendicular and angle *EFG* is indeed a right angle. This means that we can think of *EF* as the base and *FG* as the height. The length of *EF* is 10 units and the length of *FG* is 5 units. So the area of triangle *EFG* is 25 square units because $\frac{1}{2} \cdot 10 \cdot 5 = 25$.

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Practice

Coordinate Proof

1. A quadrilateral has vertices $A = (0, 0)$, $B = (1, 3)$, $C = (0, 4)$, and $D = (-1, 1)$. Prove that $ABCD$ is a parallelogram.

2. A rhombus has vertices at $(0, 0), (5, 0), (3, 4)$, and $(8, 4)$.
   a. Find the slopes of the 2 diagonals of the rhombus.

   b. What do the slopes tell you about the diagonals in this rhombus?

3. Respond to each question.
   a. Show that the triangle formed with vertices at $(0, 0), (4, 3)$, and $(-2, 11)$ is a right triangle.

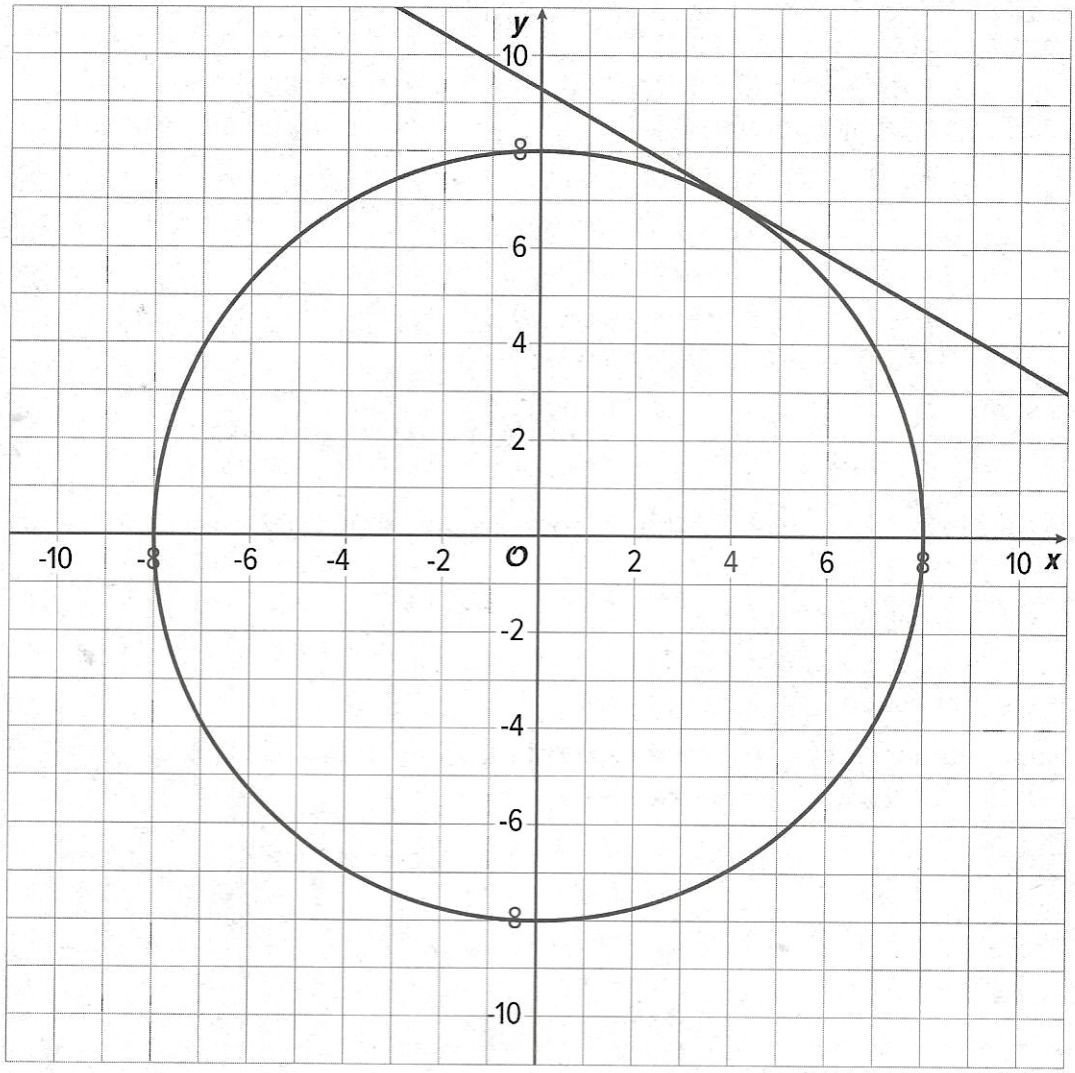
   b. Find the area of the triangle.

4. For each pair of figures, at how many points is it possible that they intersect? List all possibilities. (Lesson 6-13)
   a. two distinct lines

   b. a line and a circle

   c. a line and a parabola

5. Here are the graphs of the circle centered at (0, 0) with radius 8 and the line given by $4x + 7y = 65$. Determine whether the circle and the line intersect at the point (4, 7). (Lesson 6-13)



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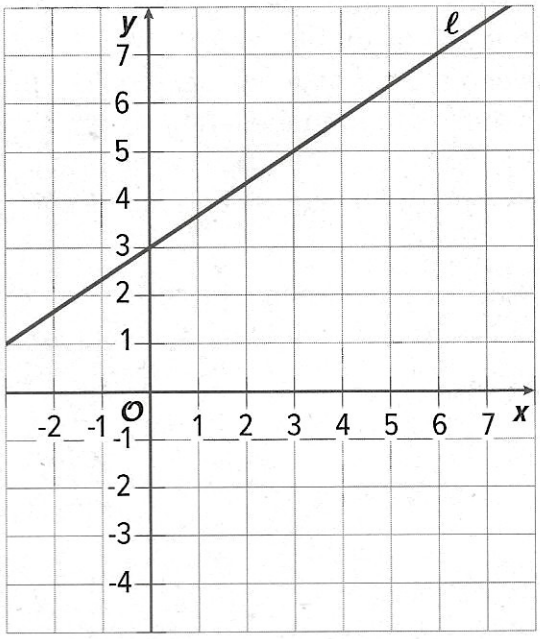
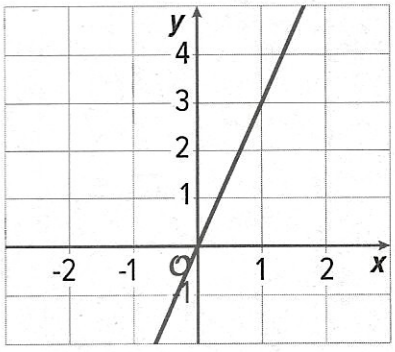
6. Here is a line ℓ. Write equations for and graph 2 different lines perpendicular to ℓ and 2 different lines parallel to ℓ. (Lesson 6-12)



7. The line shown is rotated 90 degrees clockwise around the origin. What is the slope of its image? (Lesson 6-11)

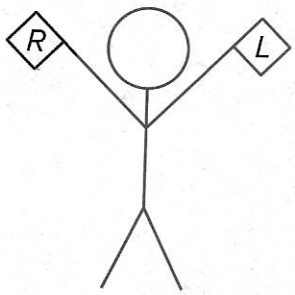


8. Elena is writing an explanation on completing the square for circle equations for Diego because he missed class. Here is what Elena wrote: (Lesson 6-6)

"Suppose we want to rewrite an equation in the form $(x - h)^2 + (y - k)^2 = r^2$. First, move any numbers to the right side and group terms with the same variables together on the left. Second, figure out what must be added to the set of x-terms and the set of y-terms to create 2 perfect square trinomials. Add those numbers to both the left and right sides of the equation. Next, rewrite the perfect square trinomials as squared binomials, and rewrite the right side in the form r^2. The center and radius can now be read directly from the equation."

Diego is unsure how to determine what needs to be added in order to get 2 perfect square trinomials. Explain how to determine the values.

9. The semaphore alphabet is a way to use flags to signal messages. Here's how to signal the letter U. Describe a transformation that would take the left hand flag to the right hand flag. (Lesson 1-13)



Lesson 6-15

Weighted Averages

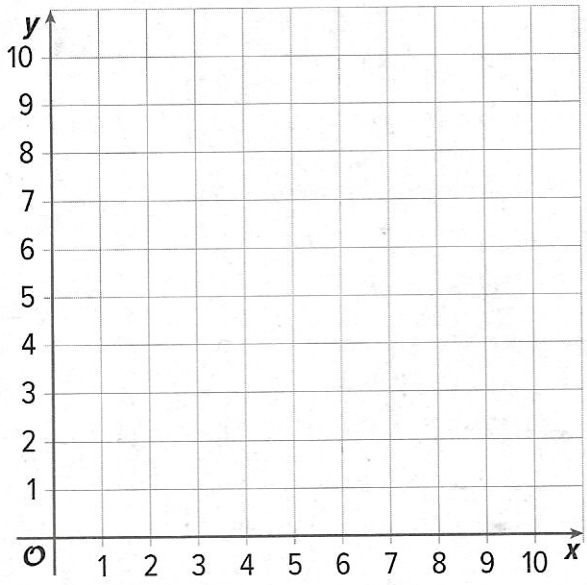
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's split segments using averages and ratios.

Warm Up

15.1 Part Way: Points

For the questions in this activity, use the coordinate grid if it is helpful to you.



1. What is the midpoint of the segment connecting (1, 2) and (5, 2)?

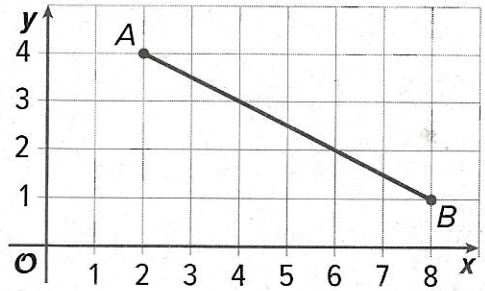
2. What is the midpoint of the segment connecting (5, 2) and (5, 10)?

3. What is the midpoint of the segment connecting (1, 2) and (5, 10)?

Activity

15.2 Part Way: Segment

Point A has coordinates (2, 4).
Point B has coordinates (8, 1).



1. Find the point that partitions segment AB in a 2 : 1 ratio.

2. Calculate $C = \frac{1}{3}A + \frac{2}{3}B$.

3. What do you notice about your answers to the first 2 questions?

4. For 2 new points K and L, write an expression for the point that partitions segment KL in a 3 : 1 ratio.

Are you ready for more?

Consider the general quadrilateral $QRST$ with $Q = (0, 0)$, $R = (a, b)$, $S = (c, d)$, and $T = (e, f)$.

1. Find the midpoints of each side of this quadrilateral.

2. Show that if these midpoints are connected consecutively, the new quadrilateral formed is a parallelogram.

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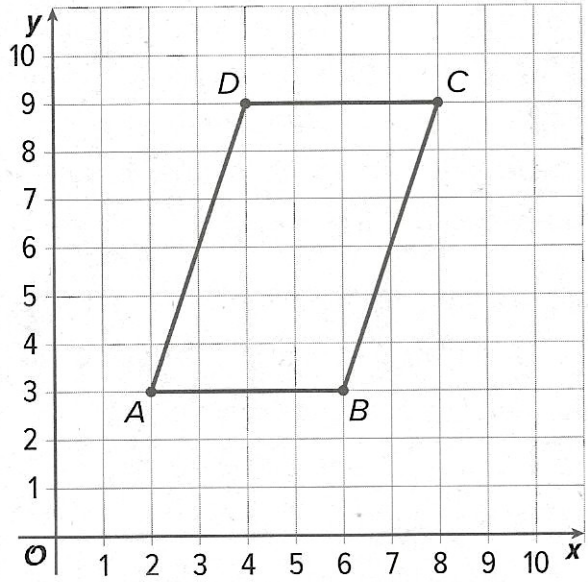
Activity

15.3 Part Way: Quadrilateral

Here is quadrilateral *ABCD*.



1. Find the point that partitions segment *AB* in a 1 : 4 ratio. Label it *B′*.

2. Find the point that partitions segment *AD* in a 1 : 4 ratio. Label it *D′*.

3. Find the point that partitions segment *AC* in a 1 : 4 ratio. Label it *C′*.

4. Is *AB′C′D* a dilation of *ABCD*? Justify your answer.

Summary

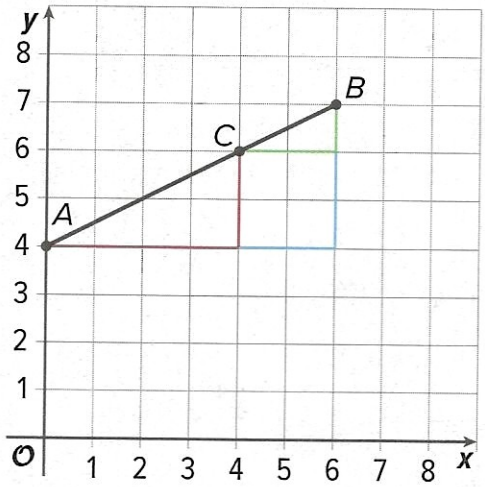
Weighted Averages

To find the midpoint of a line segment, we can average the coordinates of the endpoints. For example, to find the midpoint of the segment from $A = (0, 4)$ to $B = (6, 7)$, average the coordinates of A and B: $\left(\frac{0+6}{2}, \frac{4+7}{2}\right) = (3, 5.5)$.

Another way to write what we just did is $\frac{1}{2}(A + B)$ or $\frac{1}{2}A + \frac{1}{2}B$.

Now, let's find the point that is $\frac{2}{3}$ of the way from A to B. In other words, we'll find point C so that segments AC and CB are in a 2 : 1 ratio.

In the horizontal direction, segment AB stretches from $x = 0$ to $x = 6$. The distance from 0 to 6 is 6 units, so we calculate $\frac{2}{3}$ of 6 to get 4. Point C will be 4 horizontal units away from A, which means an x-coordinate of 4.



In the vertical direction, segment AB stretches from $y = 4$ to $y = 7$. The distance from 4 to 7 is 3 units, so we can calculate $\frac{2}{3}$ of 3 to get 2. Point C must be 2 vertical units away from A, which means a y-coordinate of 6.

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It is possible to do this all at once by saying $C = \frac{1}{3}A + \frac{2}{3}B$. This is called a weighted average. Instead of finding the point in the middle, we want to find a point closer to B than to A. So we give point B more weight—it has a coefficient of $\frac{2}{3}$ rather than $\frac{1}{2}$ as in the midpoint calculation. To calculate $C = \frac{1}{3}A + \frac{2}{3}B$, substitute and evaluate.

$$\frac{1}{3}A + \frac{2}{3}B$$

$$\frac{1}{3}(0, 4) + \frac{2}{3}(6, 7)$$

$$\left(0, \frac{4}{3}\right) + \left(4, \frac{14}{3}\right)$$

$$(4, 6)$$

Either way, we found that the coordinates of C are (4, 6).

Practice

Weighted Averages

1. Consider the parallelogram with vertices at (0, 0), (4, 0), (2, 3), and (6, 3). Where do the diagonals of this parallelogram intersect?

   (A.) (3, 1.5)

   (B.) (4, 2)

   (C.) (2, 4)

   (D.) (3.5, 3)

2. What is the midpoint of the line segment with endpoints (1, -2) and (9, 8)?

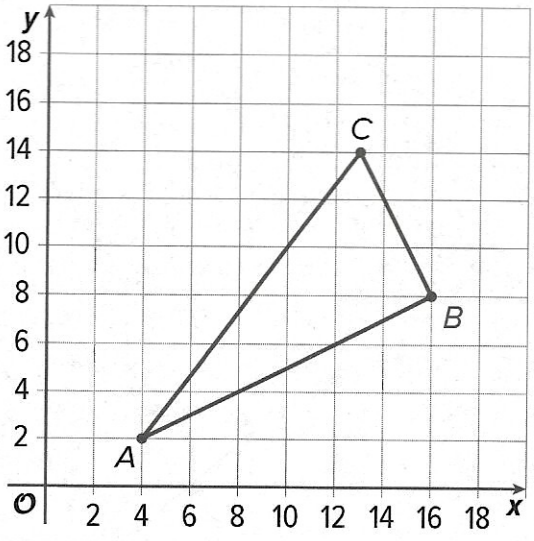
   (A.) (3, 5)

   (B.) (4, 3)

   (C.) (5, 3)

   (D.) (5, 5)

3. Graph the image of triangle *ABC* under a dilation with center *A* and scale factor $\frac{2}{3}$.



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4. A quadrilateral has vertices $A = (0, 0)$, $B = (2, 4)$, $C = (0, 5)$, and $D = (-2, 1)$. Prove that $ABCD$ is a rectangle. (Lesson 6-14)

5. A quadrilateral has vertices $A = (0, 0)$, $B = (1, 3)$, $C = (0, 4)$, and $D = (-1, 1)$. Select the most precise classification for quadrilateral $ABCD$. (Lesson 6-14)

(A.) quadrilateral

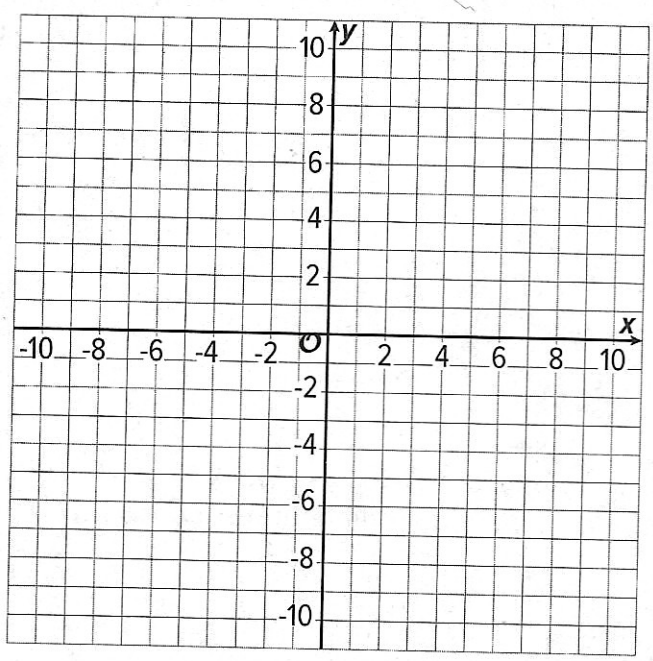
(B.) parallelogram

(C.) rectangle

(D.) square

6. Write an equation whose graph is a line perpendicular to the graph of $x = -7$ and which passes through the point $(-7, 1)$. (Lesson 6-12)

7. Graph the equations $(x + 1)^2 + (y - 1)^2 = 64$ and $y = 1$. Where do they intersect?). (Lesson 6-13)



8. A parabola has a focus of (2, 5) and a directrix of $y = 1$. Decide whether each point on the list is on this parabola. Explain your reasoning. (Lesson 6-7)

a. (-1, 5)

b. (2, 3)

c. (6, 6)

Lesson 6-16

Weighted Averages in a Triangle

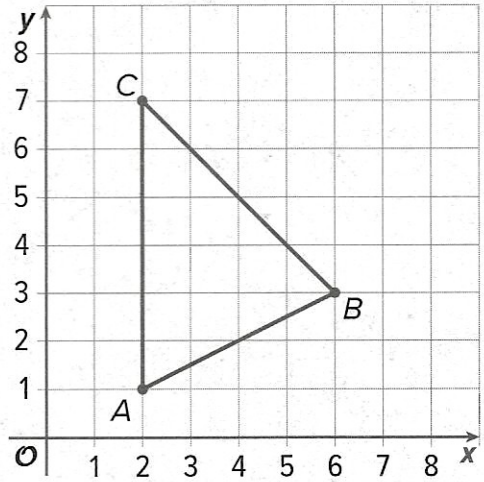
NAME _____ DATE _____ PERIOD _____

Learning Goal Let's partition special line segments in triangles.

Warm Up

16.1 Triangle Midpoints

Triangle *ABC* is graphed.



Find the midpoint of each side of this triangle.

Activity

16.2 Triangle Medians

Your teacher will tell you how to draw and label the **medians** of the triangle in the warm-up.

1. After the medians are drawn and labeled, measure all 6 segments inside the triangle using centimeters. What is the ratio of the 2 parts of each median?

2. Find the coordinates of the point that partitions segment AN in a 2 : 1 ratio.

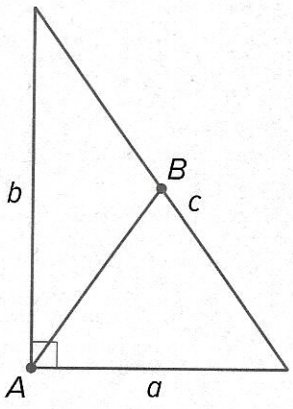
3. Find the coordinates of the point that partitions segment BL in a 2 : 1 ratio.

4. Find the coordinates of the point that partitions segment CM in a 2 : 1 ratio.

Are you ready for more?

In the image, AB is a median.

Find the length of AB in terms of a, b, and c.



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Activity

16.3 Any Triangle's Medians

The goal is to prove that the medians of any triangle intersect at a point. Suppose the vertices of a triangle are (0, 0), (w, 0), and (a, b).

1. Each student in the group should choose 1 side of the triangle. If your group has 4 people, 2 can work together. Write an expression for the midpoint of the side you chose.

2. Each student in the group should choose a median. Write an expression for the point that partitions each median in a 2 : 1 ratio from the vertex to the midpoint of the opposite side.

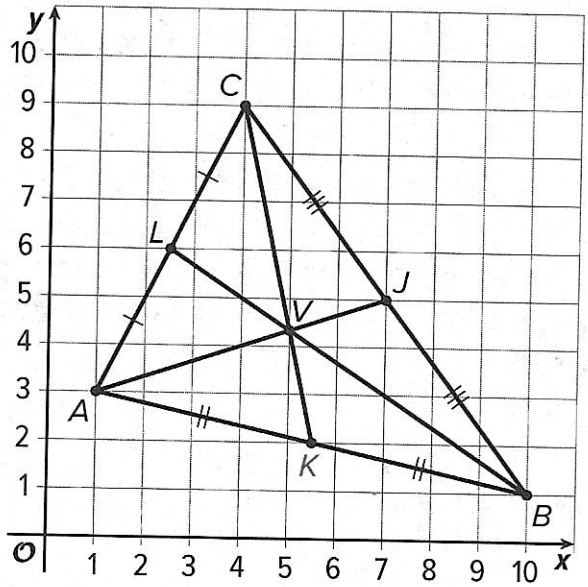
3. Compare the coordinates of the point you found to those of your groupmates. What do you notice?

4. Explain how these steps prove that the 3 medians of any triangle intersect at a single point.

Summary

Weighted Averages in a Triangle

Here is a triangle with its **medians** drawn in. A median is a line segment drawn from a vertex of a triangle to the midpoint of the opposite side. Triangles have 3 medians, with 1 for each vertex.



Notice that the medians intersect at 1 point. This point is always $\frac{2}{3}$ of the distance from a vertex to the opposite midpoint. Another way to say this is that the point of intersection, V, partitions segments AJ, BL, and CK so that the ratios $AV : VJ, BV : VL$, and $CV : VK$ are all $2 : 1$.

We can prove this by working with a general triangle that can represent any triangle. Since any triangle can be transformed so that 1 vertex is on the origin and 1 side lies on the x-axis, we can say that our general triangle has vertices $(0, 0), (0, w)$, and (a, b). Through careful calculation, we can show that all 3 medians go through the point $\left(\frac{a+w}{3}, \frac{b}{3}\right)$. Therefore, the medians intersect at this point, which partitions each median in a $2 : 1$ ratio from the vertex to the opposite side's midpoint.

Glossary

median (geometry)

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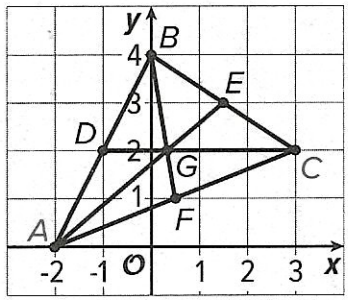
Practice

Weighted Averages in a Triangle

1. Triangle *ABC* and its medians are shown.



Select **all** statements that are true.

(A.) The medians intersect at $\left(\frac{1}{3}, 2\right)$.

(B.) The medians and altitudes are the same for this triangle.

(C.) An equation for median *AE* is $y = \frac{6}{7}(x + 2)$.

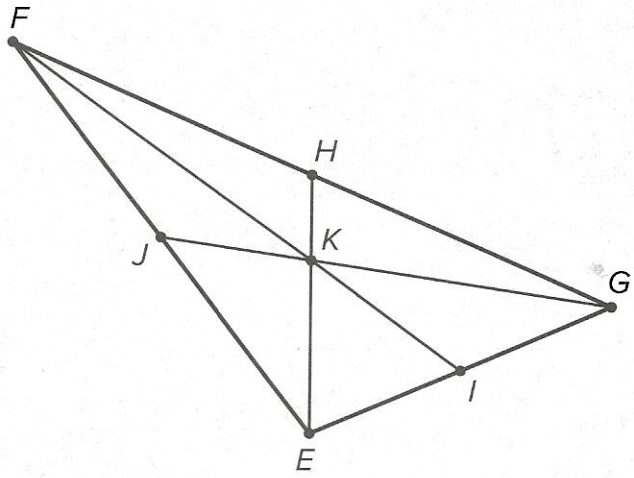
(D.) Point *G* is $\frac{2}{3}$ of the way from *A* to *E*.

(E.) Median *BF* is congruent to median *CD*.

2. Triangle *ABC* has vertices at (-2, 0), (-1, 6), and (6, 0). What is the point of intersection of the triangle's medians?

3. Triangle *EFG* and its medians are shown.

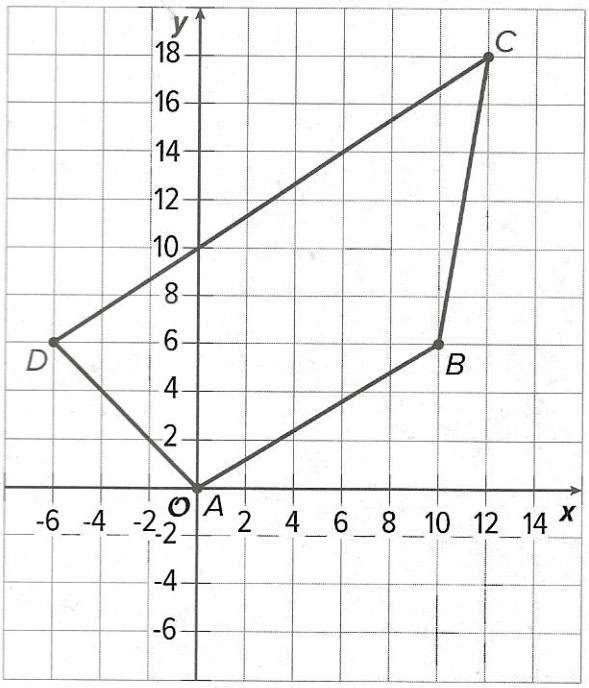
Match each pair of segments with the ratios of their lengths.



A. $GK : KJ$	**1.** 1 : 1
B. $GH : HF$	**2.** 1 : 2
C. $HK : KE$	**3.** 2 : 1

4. Given $A = (-3, 2)$ and $B = (7, -10)$, find the point that partitions segment AB in a 1 : 4 ratio. **(Lesson 6-15)**

5. Graph the image of quadrilateral *ABCD* under a dilation using center *A* and scale factor $\frac{1}{3}$. **(Lesson 6-15)**



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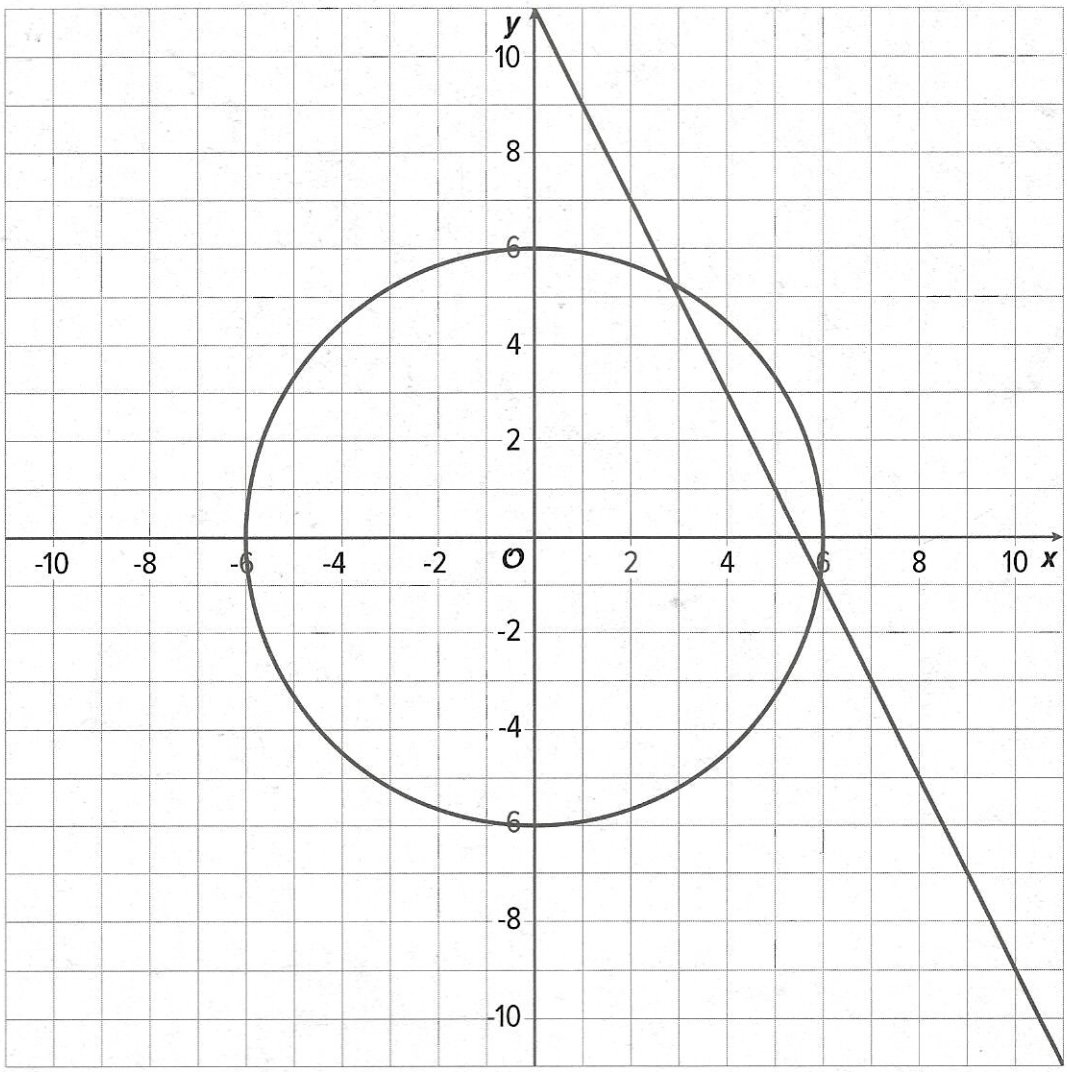
6. A trapezoid is a quadrilateral with at least one pair of parallel sides. Show that the quadrilateral formed by the vertices (0, 0), (5, 2), (10, 10), and (0, 6) is a trapezoid. (Lesson 6-14)

7. Here are the graphs of the circle centered at (0, 0) with radius 6 units and the line given by $2x + y = 11$. Determine whether the circle and the line intersect at the point (3, 5). Explain or show your reasoning. (Lesson 6-13)



8. A parabola has focus (-3, 2) and directrix $y = -3$. The point $(a, 5)$ is on the parabola. How far is this point from the focus? **(Lesson 6-8)**

(A.) 8 units

(B.) 5 units

(C.) 3 units

(D.) 2 units

Lesson 6-17

Lines in Triangles

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Learning Goal Let's investigate more special segments in triangles.

Warm Up

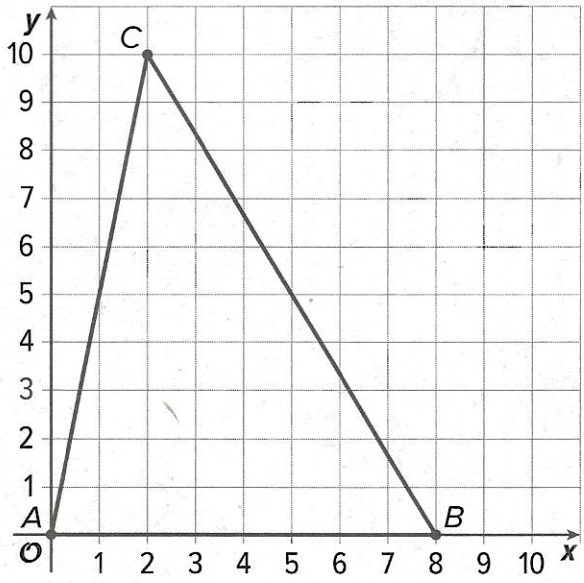
17.1 Folding Altitudes

Draw a triangle on tracing paper. Fold the altitude from each vertex.

Activity

17.2 Altitude Attributes

Triangle *ABC* is graphed.



1. Find the slope of each side of the triangle.

2. Find the slope of each altitude of the triangle.

3. Sketch the altitudes. Label the point of intersection *H*.

4. Write equations for all 3 altitudes.

5. Use the equations to find the coordinates of H and verify algebraically that the altitudes all intersect at H.

Are you ready for more?

Any triangle ABC can be translated, rotated, and dilated so that the image A' lies on the origin, B' lies on the point (1,0), and C' has position (a, b). Use this as a starting point to prove that the altitudes of all triangles all meet at the same point.

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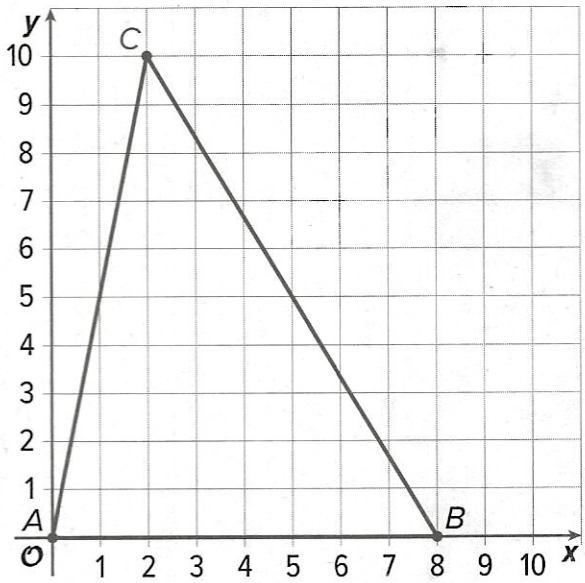
Activity

17.3 Percolating on Perpendicular Bisectors

Triangle *ABC* is graphed.



1. Find the midpoint of each side of the triangle.

2. Sketch the perpendicular bisectors, using an index card to help draw 90 degree angles. Label the intersection point *P*.

3. Write equations for all 3 perpendicular bisectors.

4. Use the equations to find the coordinates of *P* and verify algebraically that the perpendicular bisectors all intersect at *P*.

Activity

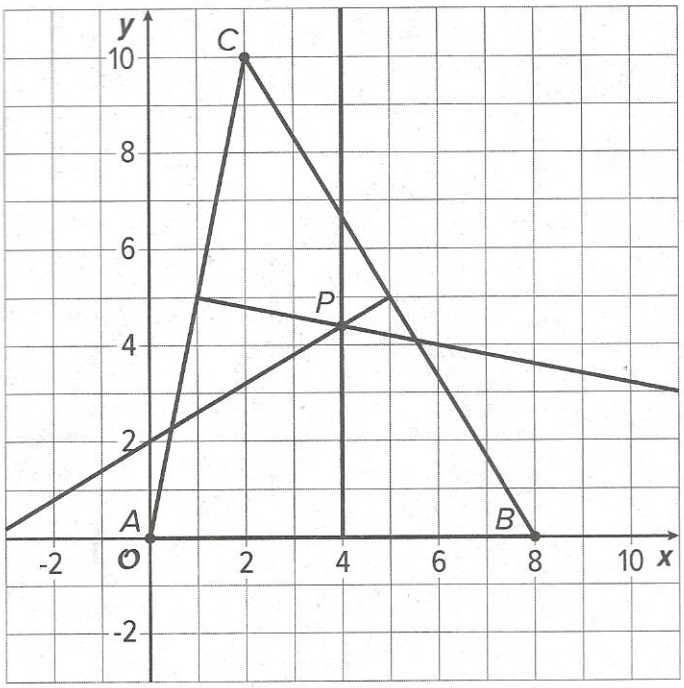
17.4 Perks of Perpendicular Bisectors

Consider triangle *ABC* from an earlier activity.

1. What is the distance from *A* to *P*, the intersection point of the perpendicular bisectors of the triangle's sides? Round to the nearest tenth.

2. Write the equation of a circle with center *P* and radius *AP*.

3. Construct the circle. What do you notice?



4. Verify your hypothesis algebraically.

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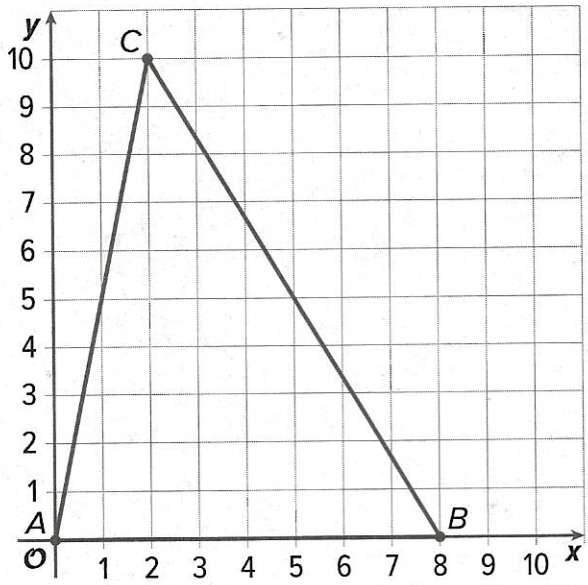
Activity

17.5 Amazing Points

Consider triangle *ABC* from earlier activities.



1. Plot point *H*, the intersection point of the altitudes.

2. Plot point *P*, the intersection point of the perpendicular bisectors.

3. Find the point where the 3 medians of the triangle intersect. Plot this point and label it *J*.

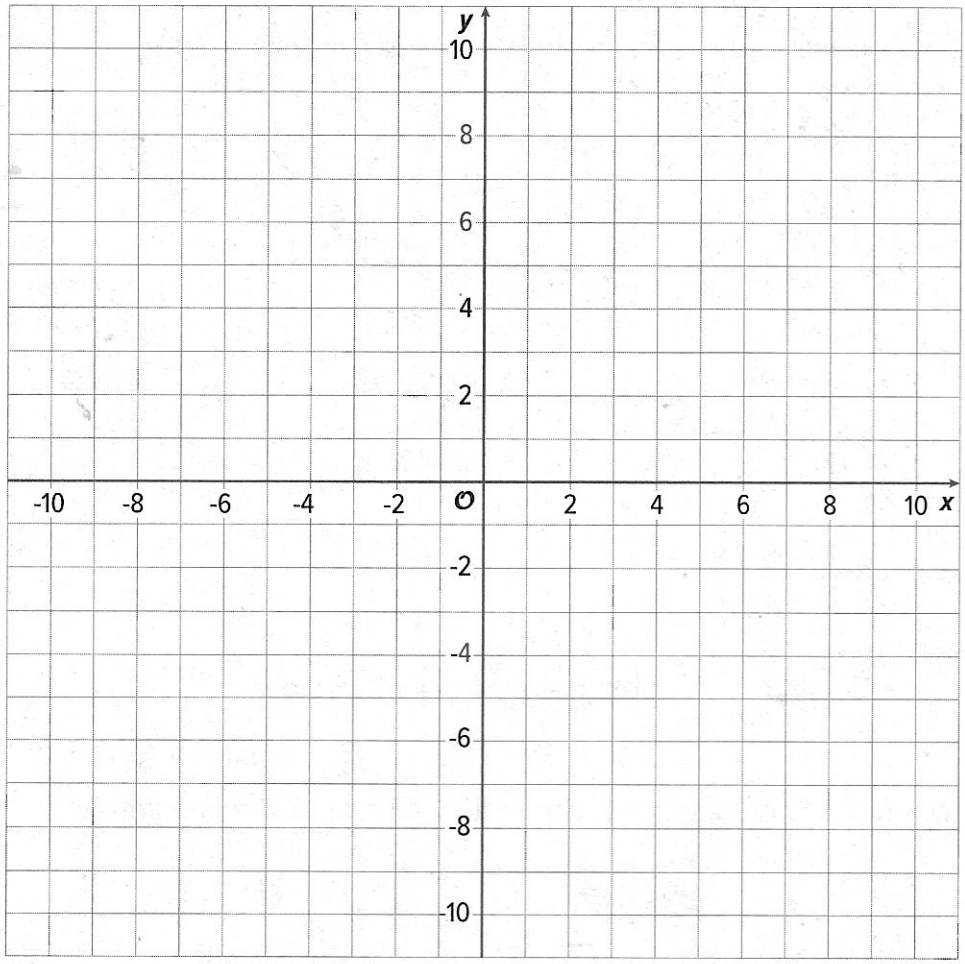
4. What seems to be true about points *H*, *P*, and *J*? Prove that your observation is true.

Activity

17.6 Tiling the (Coordinate) Plane

A tessellation covers the entire plane with shapes that do not overlap or leave gaps.

1. Tile the plane with congruent rectangles:
   a. Draw the rectangles on your grid.
   b. Write the equations for lines that outline 1 rectangle.



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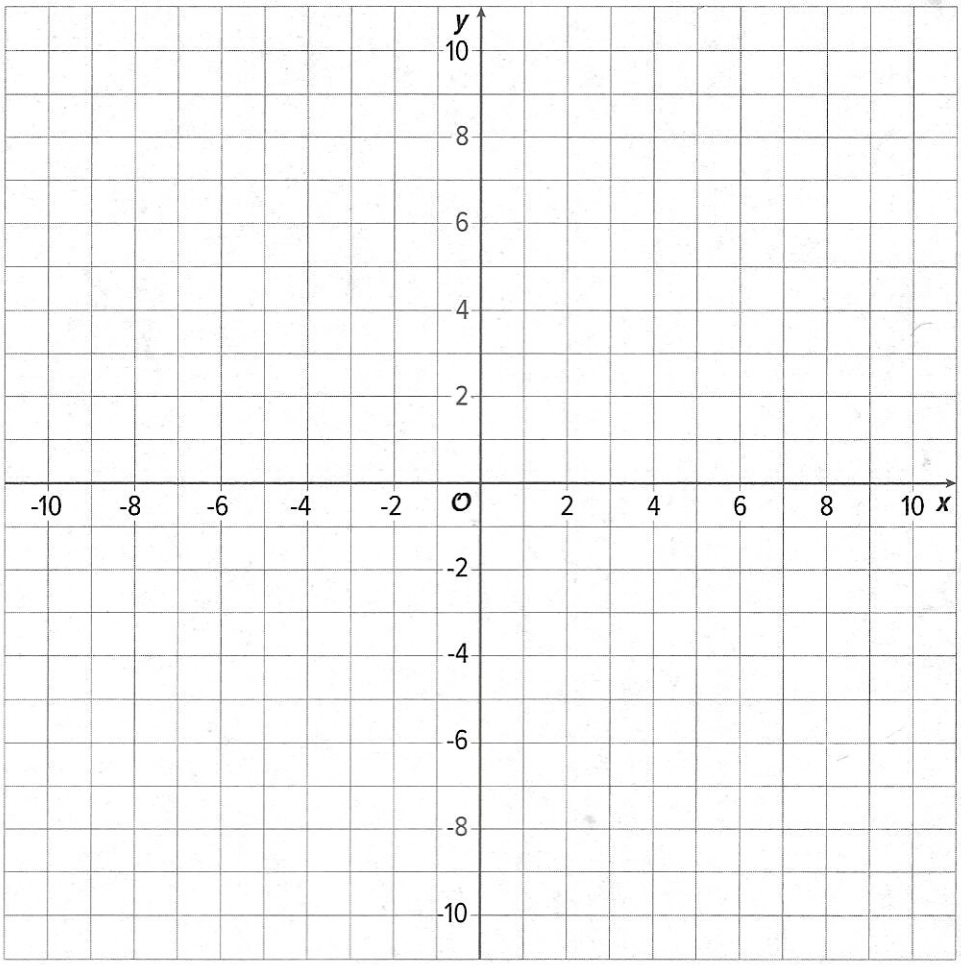
2. Tile the plane with congruent right triangles:

   a. Draw the right triangles on your grid.

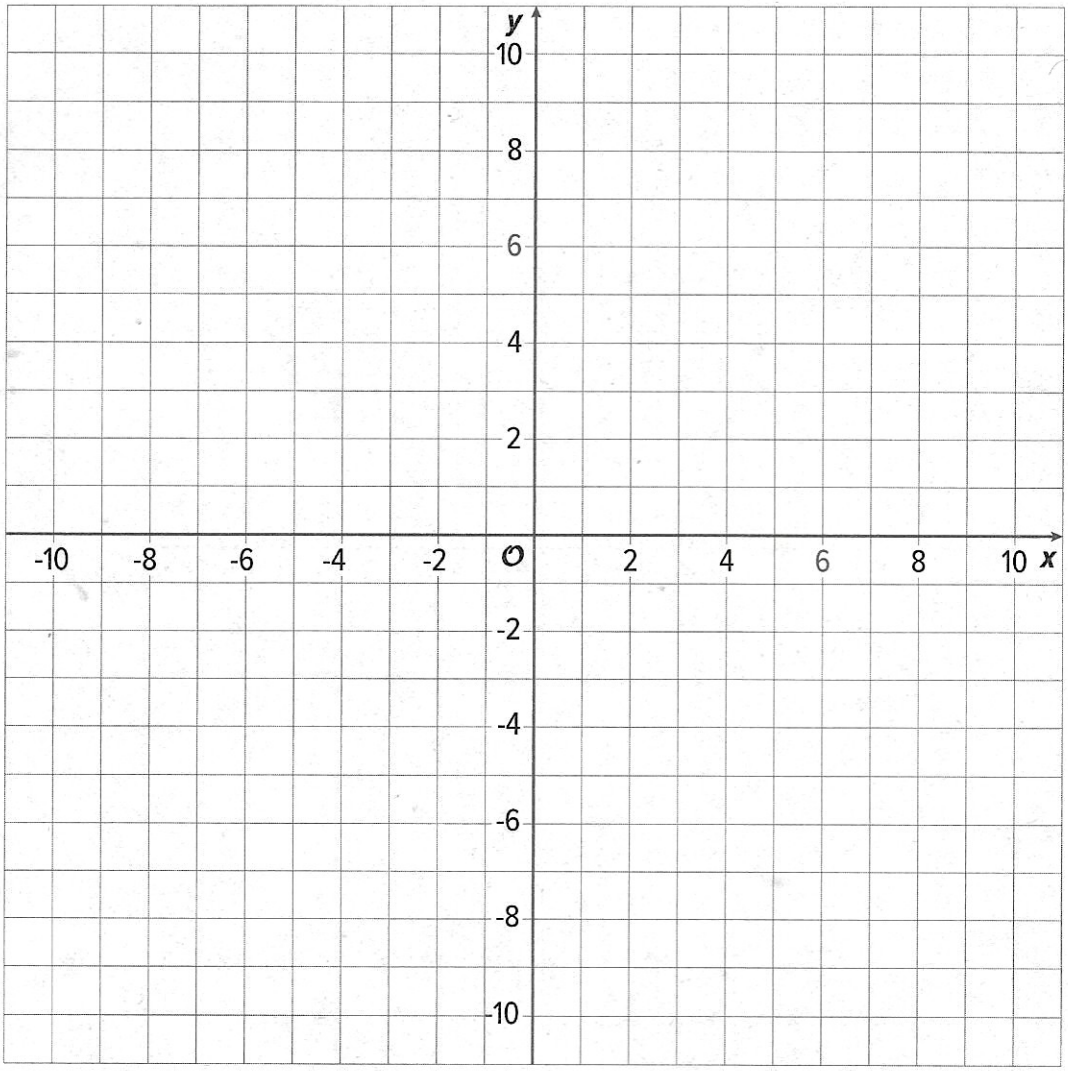
   b. Write the equations for lines that outline 1 right triangle.



3. Tile the plane with any other shapes:

   a. Draw the shapes on your grid.

   b. Write the equations for lines that outline 1 of the shapes.



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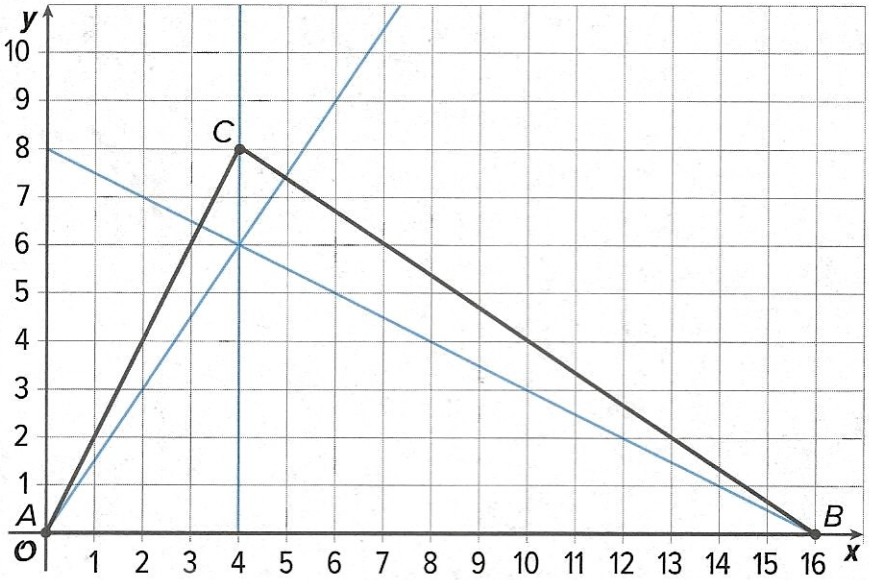
Summary

Lines in Triangles

The 3 medians of a triangle always intersect in 1 point. We can use coordinate geometry to show that the altitudes of a triangle intersect in 1 point, too. The 3 altitudes of triangle *ABC* are shown here. They appear to intersect at the point (4, 6). By finding their equations, we can prove this is true.



The slopes of sides *AB*, *BC*, and *AC* are 0, $-\frac{2}{3}$, and 2. The altitude from *C* is the vertical line $x = 4$. The slope of the altitude from *A* is $\frac{3}{2}$. Since the altitude goes through (0, 0), its equation is $y = \frac{3}{2}x$. The slope of the altitude from *B* is $-\frac{1}{2}$. Following this slope over to the *y*-axis we can see that the *y*-intercept is 8. So the equation for this altitude is $y = -\frac{1}{2}x + 8$.

We can now verify that (4, 6) lies on all 3 altitudes by showing that the point satisfies the 3 equations. By substitution we see that each equation is true when $x = 4$ and $y = 6$.

Practice

Lines in Triangles

1. The 3 lines $x = 3$, $y - 2.5 = -\frac{1}{5}(x - 0.5)$, and $y - 2.5 = x - 3.5$ intersect at point P. Find the coordinates of P. Verify algebraically that the lines all intersect at P.

2. Triangle ABC has vertices at (0, 0), (5, 5), and (10, 1). Kiran calculates the point of intersection of the medians using the following steps: (Lesson 6-16)

   a. Draw the triangle.

   b. Calculate the midpoint of each side.

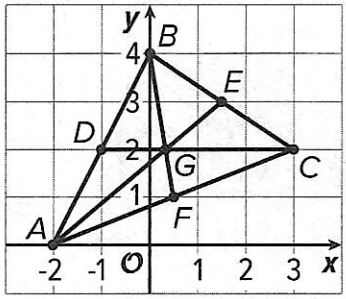
   c. Draw the medians.

   d. Write an equation for 2 of the medians.

   e. Solve the system of equations.

   Use Kiran's method to calculate the point of intersection of the medians.

3. Triangle ABC and its medians are shown. Write an equation for median AE. (Lesson 6-16)



4. Given $A = (1, 2)$ and $B = (7, 14)$, find the point that partitions segment AB in a 2 : 1 ratio. (Lesson 6-15)

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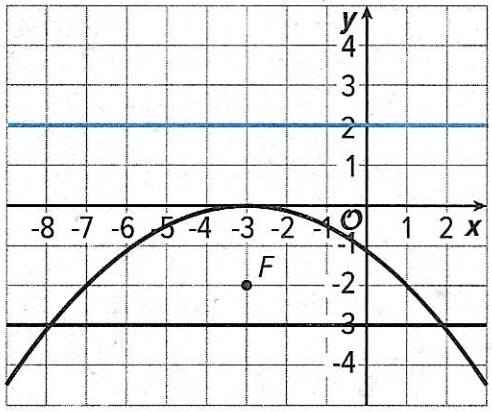
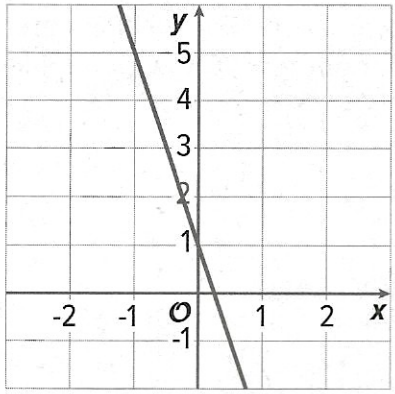
5. A quadrilateral has vertices $A = (0, 0)$, $B = (4, 6)$, $C = (0, 12)$, and $D = (-4, 6)$ Mai thinks the quadrilateral is a rhombus and Elena thinks the quadrilateral is a square. Do you agree with either of them? Show or explain your reasoning. **(Lesson 6-14)**

6. The image shows a graph of the parabola with focus (-3, -2) and directrix $y = 2$, and the line given by $y = -3$. Find and verify the points where the parabola and the line intersect. **(Lesson 6-13)**



7. For each equation, is the graph of the equation parallel to the line shown, perpendicular to the line shown, or neither? (Lesson 6-12)



a. $y = 0.25x$

b. $y = 2x - 4$

c. $y - 2 = -4(x-3)$

d. $2y + 8x = 7$

e. $x - 4y = 3$

8. Write 2 equivalent equations for a line with x-intercept (3, 0) and y-intercept (0, 2) (Lesson 6-9)

9. Parabola A and parabola B both have the line $y = -2$ as the directrix. Parabola A has its focus at (3, 4) and parabola B has its focus at (5, 0). Select **all** true statements. (Lesson 6-7)

(A.) Parabola A is wider than parabola B.

(B.) Parabola B is wider than parabola A.

(C.) The parabolas have the same line of symmetry.

(D.) The line of symmetry of parabola A is to the right of that of parabola B.

(E.) The line of symmetry of parabola B is to the right of that of parabola A.

Learning Targets

Lesson	Learning Target(s)
6-1 Rigid Transformations in the Plane	• I can prove triangles are congruent using coordinates. • I can reflect, rotate, and translate figures in the coordinate plane.
6-2 Transformations as Functions	• I can use coordinate transformation notation to take points in the plane as inputs and give other points as outputs.
6-3 Types of Transformations	• I can determine whether a transformation produces congruent or similar images (or neither).
6-4 Distances and Circles	• I can derive an equation for a circle in the coordinate plane.

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Lesson	Learning Target(s)
6-5 Squares and Circles	• I understand how squared binomials relate to the equation of a circle.
6-6 Completing the Square	• I can complete the square to find the center and radius of a circle.
6-7 Distances and Parabolas	• I know that a parabola is the set of points equidistant from a given point and line.
6-8 Equations and Graphs	• I can derive an equation for a parabola in the coordinate plane given a focus and a directrix.
6-9 Equations of Lines	• I can use the definition of slope to write the equation for a line in point-slope form.

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Lesson	Learning Target(s)
6-10 Parallel Lines in the Plane	• I can prove that the slopes of parallel lines are equal. • I can use slopes of parallel lines to solve problems.
6-11 Perpendicular Lines in the Plane	• I can prove that the slopes of perpendicular lines are opposite reciprocals. • I can use slopes of perpendicular lines to solve problems.
6-12 It's All on the Line	• I can gather information about a line and write its equation.
6-13 Intersection Points	• I can use a graph to find the intersection points of a line and a circle.
6-14 Coordinate Proof	• I can use coordinates of figures to prove geometric theorems.

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Lesson	Learning Target(s)
6-15 Weighted Averages	• I can calculate the coordinates of a point on a line segment that partitions the segment in a given ratio.
6-16 Weighted Averages in a Triangle	• I can determine the point where the medians of a triangle intersect.
6-17 Lines in Triangles	• I can determine the point where the altitudes of a triangle intersect.

Notes:

TO BE CONTINUED
IN NEXT UNIT

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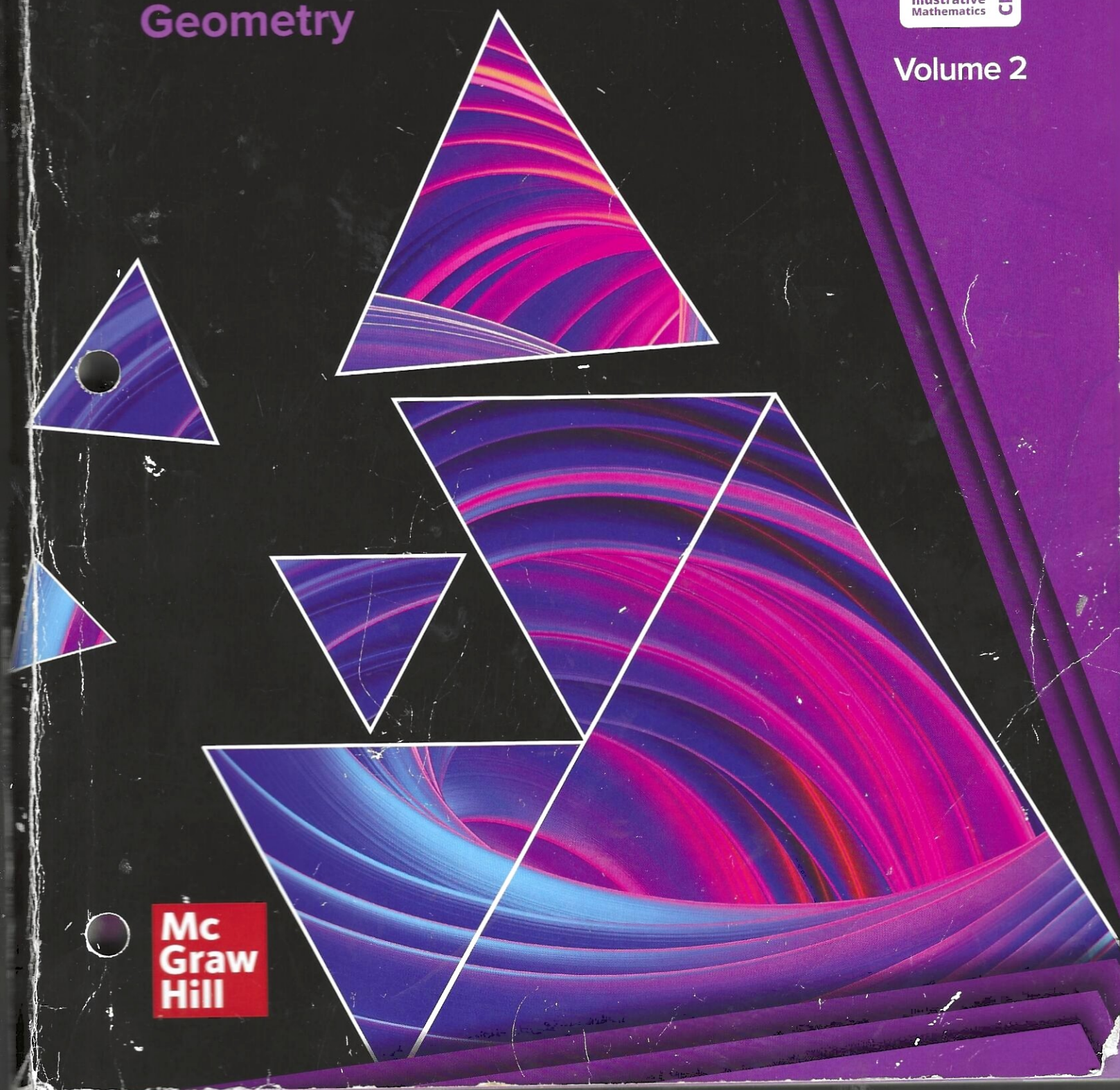

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